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Interaction effects may actually be nonlinear effects in disguise: A review of the problem and potential solutions

William C.M. Belzak*, Daniel J. Bauer

Department of Psychology and Neuroscience, University of North Carolina at Chapel Hill, United States

HIGHLIGHTS

- Nonlinear effects are rarely tested in the behavioral sciences.
- Interactions can masquerade for nonlinear effects when the predictors are correlated.
- Previously described for regression, the same problem occurs with moderated mediation.
- We explicate these statistical issues using graphs, algebra, and empirical examples.
- We recommend ways to empirically distinguish interactions from nonlinear effects.

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ABSTRACT

It is common in addictions research for statistical analyses to include interaction effects to test moderation hypotheses. Far less commonly do researchers consider the possibility that a given predictor may exert a nonlinear effect on the outcome. This lack of attention to the possible nonlinear effects of individual predictors is problematic because it may result in identification of entirely spurious interactions with other, correlated predictors. Given the commonplace practice of testing interactions, and the rarity of testing nonlinear effects, we speculate that some of the significant interactions reported in the literature may actually be spurious, reflecting only the misspecification of nonlinear effects. We outline the mathematical reasons for this problem using the relatively simple case of a quadratic regression model. Within this context, prior research by Busemeyer and Jones (1983) clearly demonstrated that quadratic effects of individual predictors can masquerade as interaction effects between correlated predictors. Furthermore, the explosive growth of mediation, moderation, and moderated mediation analyses in behavioral research makes this issue especially relevant for researchers of addiction. In this article, we (1) call further attention to the potential problems of omitting nonlinear effects in linear regression, (2) extend these findings to the more complex moderated mediation model, and (3) provide practical recommendations for applied researchers for differentiating nonlinear from interactive effects.

Tests of hypotheses concerning interaction effects, in which the effect of one predictor depends on the value of another, are ubiquitous in the behavioral, health, and social sciences. As a case in point, Fig. 1 shows that, over the past 20 years, approximately one third of publications in *Addictive Behaviors* has included at least one interaction effect.[1] Further, this proportion appears to have increased modestly over time. In one example, Woo, Wang, and Tran (2017) found an interaction between psychological distress and ethnicity in the prediction of binge drinking, with a stronger effect for Asian Americans than for Caucasian Americans.

It is far less common to see researchers consider whether an individual predictor might exert a nonlinear effect on the outcome (including quadratic, or curvilinear effects). Indeed, Fig. 1 shows that less than 3% of articles published in *Addictive Behaviors* over the past

* Corresponding author.
E-mail address: wbelzak@live.unc.edu (W.C.M. Belzak).

[1] We extracted all research articles published in *Addictive Behaviors* from 1998 to 2017 (i.e., Volumes 23–75) using the Elsevier API and searched for key words, "interaction", "interacts", "interacted", "moderation", "moderates", and "moderated", in the results sections. We counted a total of 3641 articles published during the period (i.e., excluding articles that did not include results sections, such as editorial letters and some review articles) and found 1129 articles that included at least one of these key words (i.e., 31%). Given the potential mismatch between keywords in the results sections and actual tests of interaction effects, we caution readers that this percentage is an approximation.

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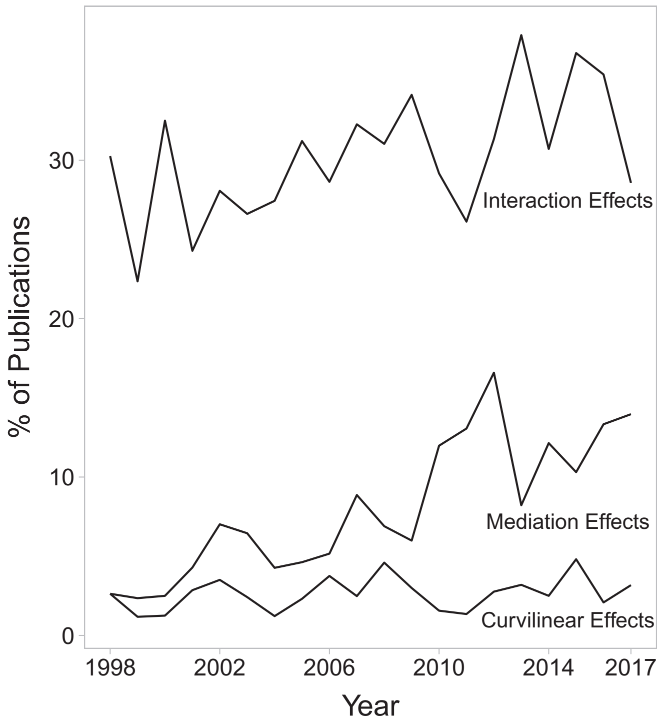


Fig. 1. Percentage of articles published each year in *Addictive Behaviors* from 1998 to 2017 with interaction, mediation, or curvilinear effects.

20 years tested for a nonlinear relationship of this kind.[2] In one example, Parnes, Rahm-Knigge, and Conner (2017) specified a quadratic relationship between sexual orientation and substance use among women. Women identifying as having mixed sexual orientation were more likely to engage in substance use than women who were at either end of the sexual orientation continuum.

While it is clear the number of interaction effects tested in addictions research dwarfs the number of nonlinear relationships, it is not entirely clear why. One possible reason is that most effects are truly linear, making the testing of nonlinear effects unnecessary. A more likely explanation is that interaction effects are simply more salient to researchers than nonlinear effects, despite both being relatively common (Cortina, 1993). Researchers may be primarily concerned with establishing whether an effect exists and under what conditions, questions naturally addressed by interaction models. Less obvious may be the need to consider whether the function describing the relationship between a predictor and outcome is straight or curved. What researchers may not realize is that failure to attend to possible nonlinear effects may result in the false detection of interaction effects.

Busemeyer and Jones (1983) were the first to show how an interaction effect could statistically masquerade for a nonlinear relationship, focusing specifically on a quadratic relationship. They showed that failing to account for the true quadratic effect of a single predictor could lead to the identification of a false interaction effect for this predictor with another, correlated predictor. In the 35 years since this publication, other researchers have proffered various recommendations for differentiating interaction effects from nonlinear (quadratic) effects (Ganzach, 1997; Lubinski & Humphreys, 1990; MacCallum & Mar, 1995). Despite this methodological work, however, we believe many applied researchers either do not fully understand the possibility of confusing a nonlinear relationship with an interaction effect or are unaware of the implications. We found just one article out of over 300 of those published in *Addictive Behaviors* during 2017 that ruled out a quadratic effect when testing for an interaction, and this was prompted by a reviewer's request (see footnote in Simons et al., 2017). By implication, an unknown but likely non-trivial proportion of interaction effects reported in the literature may well be spurious, reflecting the omission of nonlinear effects from the fitted models.

Our goal here is to explicate this underappreciated problem and make practical recommendations for addressing it. We first review prior research showing how interaction and nonlinear effects can masquerade for one another in the linear regression model. Although linear regression provides a familiar context within which to introduce the problem, we go on to demonstrate that it is also equally relevant for more complex models. In particular, we provide new results showing how moderated mediation analysis is affected by the presence of unmodeled nonlinear effects. We chose moderated mediation given the increasing prominence of mediation analysis in addiction science and the lack of prior work considering how nonlinear effects might impact these analyses (although see Hayes & Preacher, 2010, for discussion). Throughout, similar to Busemeyer and Jones (1983), we focus on nonlinear effects that can be described by a quadratic function. The quadratic function has the virtues of mathematical simplicity and relatively common implementation compared to other nonlinear functions (see Cohen, Cohen, West, & Aiken, 2013). However, the conclusions we draw are generally applicable for other nonlinear functions. Finally, we close by discussing proposed solutions for differentiating nonlinear from interactive effects, ending with a series of practical recommendations for applied researchers. These recommendations update previous guidance from MacCallum and Mar (1995) to reflect methodological advances that have taken place since their original work.

1. Interaction effects and quadratic effects in linear regression

In this section, we describe the typical specification of interaction effects and quadratic effects in the multiple regression model. We subsequently show how one effect can masquerade for the other if the fitted model is not specified correctly.

1.1. Interaction effects

In linear regression, interactions are evaluated by entering the predictors as well as their product term into the fitted model (Cohen, 1978). With two predictors, the corresponding population-level model equation may be written as:

$$Y_i = \beta_0 + \beta_1 X_i + \beta_2 Z_i + \beta_3 (X_i \times Z_i) + \epsilon_i, \quad (1)$$

where Y_i is the outcome variable value for person i; β_0 is the intercept (expected value of Y when X and Z are simultaneously zero); β_1 is the effect of X when Z is zero; β_2 is the effect of Z when X is zero; β_3 is the interaction effect, conveying the extent to which the effect of X depends on Z or vice versa; and ϵ_i is the residual, representing the discrepancy between the predicted and actual value of Y. Note that the interpretation of the lower-order terms β_0, β_1, and β_2 all depend on the zero-point of certain predictors. As such, mean-centering continuous predictors is often recommended to facilitate interpretation (Aiken & West, 1991). For example, if Z is centered (giving it a mean of zero) then β_1 is the effect of X at the mean of Z.

Although there is no mathematical distinction between X and Z in Eq. (1), researchers often make a theoretical distinction between them, considering one to be the focal predictor and the other to be the moderator. The focal predictor is usually the predictor with the effect of primary interest. In contrast, the moderator is what changes the effect of the focal predictor on the outcome. With this distinction in mind, the

[2] We conducted a search similar to the one described in footnote 1 and used key words, "quadratic", "curvilinear", "nonlinear", "exponential", "exponentially", "logarithmic", and "logarithmically". We found 99 out of 3641 articles that included at least one of these key words in the results section (i.e., 3%). As in the case of finding interaction effects, the percentage is an approximation.

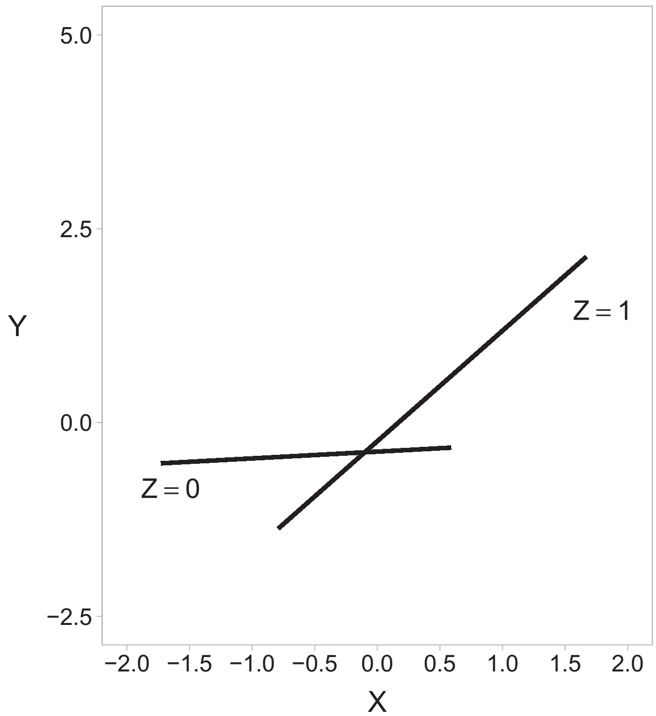


Fig. 2. Simple slopes analysis for an interaction effect. X has a mean of zero, and Z is coded 0 or 1 to indicate group membership. The length of the lines correspond to the range of observed data on X for each group defined by Z.

terms of the model can be rearranged to highlight that the effect of the focal predictor depends on the level of the moderator, as follows:

$$Y_i = (\beta_0 + \beta_2 Z_i) + (\beta_1 + \beta_3 Z_i)X_i + \epsilon_i. \quad (2)$$

Note that, at any given level of Z, Y is linearly related to X but both the intercept of the line (first set of parentheses) and its slope (second set of parentheses) depend on the value of Z. To further understand the nature of the interaction, it is common practice to compute the "simple intercept" and "simple slope" for the line describing the relationship of Y to X at different values of Z (Aiken & West, 1991; Allison, 1977; Jaccard & Turrisi, 2003). Fig. 2 plots an example of a simple slopes analysis where the focal predictor X is continuous and the moderator Z is dichotomous.

1.2. Quadratic effects

Now let us consider the specification of quadratic effects within the linear regression model.[3] Here, the squared values of the predictor are computed and entered as a second variable alongside the original (unsquared) version as shown:

$$Y_i = \beta_0 + \beta_1 X_i + \beta_2 X_i^2 + \epsilon_i. \quad (3)$$

Here, the intercept, β_0, indicates the expected value of Y when X is zero; the linear coefficient, β_1, gives the effect of X on Y when X is zero (the slope of the tangent line to the curve at $X = 0$); and the quadratic coefficient, β_2, captures the extent to which the effect of X on Y changes as X increases (the degree of curvature). Since β_0 and β_1 both depend on the zero-point of X, mean-centering X is again often recommended to facilitate interpretation.

One way to think about a quadratic effect is as an interaction of a predictor with itself. That is, the effect of the predictor varies as a function of its own level. As the predictor changes in value, the effect of the predictor on the outcome may become stronger or weaker, plateau or trough, or even change direction within the observed range of the data. We can see this by rearranging the terms of the quadratic model as we did for the interaction model:

$$Y_i = \beta_0 + (\beta_1 + \beta_2 X_i)X_i + \epsilon_i. \quad (4)$$

Within the parentheses is the effect of X, which is a linear function of its own value. This implies that the effect of X smoothly increases or decreases in magnitude across its range, yielding either a U- or inverse U-shaped function for the predictive relationship, only part of which may be present within the observed data (e.g., the first half of a U or inverse-U might reasonably approximate a curve of diminishing returns). Fig. 3 provides one example of a quadratic effect.

1.3. How one effect can masquerade for the other

The problem we wish to highlight in this paper is that a quadratic effect can masquerade as an interaction effect (or vice versa) due to their mathematical similarity. Consider the interaction and quadratic models side-by-side, with Z included in both models:

$$Y_i = \beta_0 + \beta_1 X_i + \beta_2 Z_i + \beta_3 (X_i \times Z_i) + \epsilon_i; \quad (5)$$

$$Y_i = \beta_0 + \beta_1 X_i + \beta_2 Z_i + \beta_3 (X_i \times X_i) + \epsilon_i. \quad (6)$$

Having expanded the quadratic term, notice that these equations look nearly identical. The only difference between these equations is the interaction term $X \times Z$ versus the quadratic term $X \times X$.[4] The structural similarity between these equations intimates how a quadratic effect could be conflated with an interaction effect. As the correlation between X and Z increases, the correlation between $X \times Z$ and $X \times X$ will also increase, allowing one to serve as an imperfect proxy for the other. In the limit, if the two predictors were perfectly correlated, then the two equations above would actually become mathematically identical.[5]

The implication is that if only one of the two effects – interaction or quadratic – is included in the fitted model and it is the wrong one, then the researcher may incorrectly conclude that this effect exists when it does not. Because it is much more common to fit models with interaction effects, the greatest risk is for omitted nonlinear effects to result in spurious interactions. Specifically, if X has a quadratic effect on Y but the fitted model excludes X^2 and instead includes $X \times Z$, then the product term would stand in for the squared term to the extent that X and Z are correlated, potentially leading to the false identification of an interaction effect when none is actually present. Thus, the scenario of greatest concern is that (a) there exists a true nonlinear relationship between a predictor and outcome; (b) there is a nonzero correlation between this predictor and another; and (c) a researcher tests for and finds a significant interaction effect for these two predictors without considering whether either predictor might exert a nonlinear effect on the outcome.

Fig. 4 provides a visual illustration of how this might occur. We simulated data with a true quadratic effect (i.e., Eq. (3)), fitted an interaction model to the data, and overlaid the predictions of the true quadratic model with the false interaction model. To make the results easier to visualize, we made Z a binary grouping variable, made the effect of Z zero, and induced a correlation between X and Z by having the mean of X differ between the two groups. The dashed vertical lines show the mean of X for each group, and the full distribution of X for each group is shown at the top of the plot. Note that for the group $Z = 0$, the majority of values for X are low. Over this range of X the

[3] The terms quadratic, second-order polynomial, and curvilinear are used interchangeably throughout the article.

[4] Although we have focused on X as the predictor with a potentially nonlinear effect, the same pattern would arise if it were Z instead.

[5] Given complete confounding of X and Z, however, it would not be possible to obtain separate estimates for their lower-order effects.

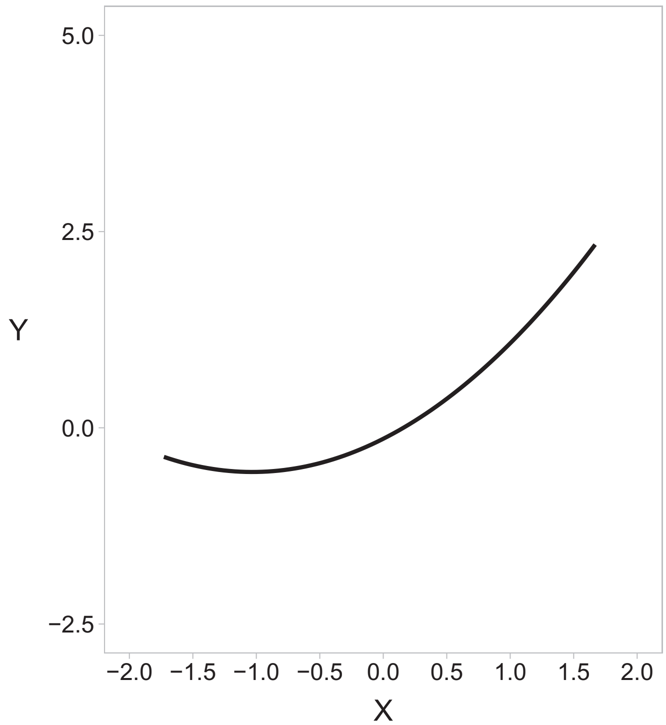


Fig. 3. A quadratic effect model. *X* has a mean of zero.

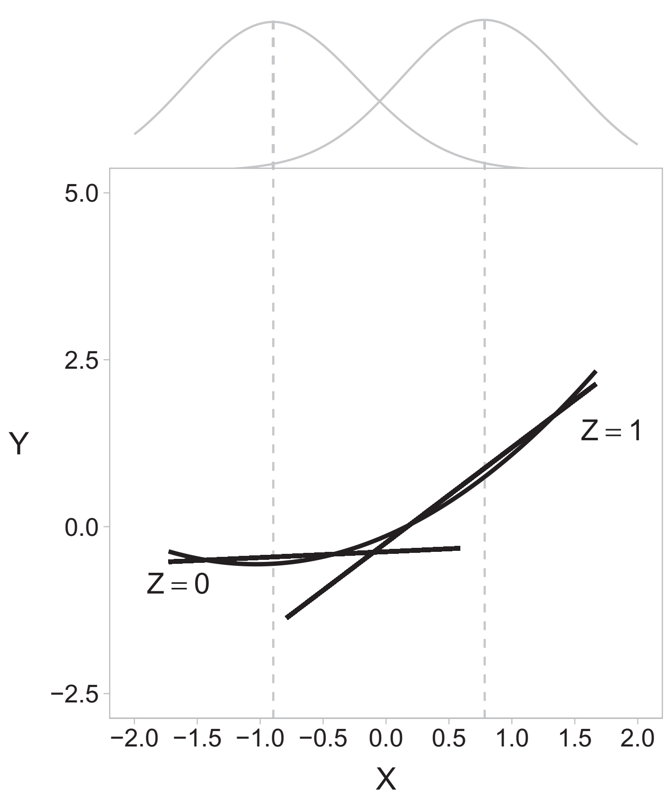


Fig. 4. Comparison of model-implied predicted values given the correct quadratic model (curved line) and incorrect interaction model (straight lines). This figure shows that, when the mean of *X* differs by *Z* (there is a correlation between *X* and *Z*), the interaction model linearly approximates different spans of the quadratic curve for each value of *Z*, producing a spurious interaction effect.

quadratic curve is at its nadir, so the interaction model implies a nearly flat line for this group. For the group $Z = 1$, however, most of the values of *X* are high and in the span where the quadratic curve is accelerating upwards. For this group, the interaction model therefore produces a steeply increasing line. Thus, fitting the interaction model alone, we would falsely conclude that the effect of *X* differs across groups, having a strong effect only when $Z = 1$. In fact, *X* has the very same (nonlinear) effect in both groups, irrespective of the value of *Z* (which just happens to be correlated with *X*).

1.4. Some mathematical results

In some cases, we can actually derive the regression coefficients for the incorrect interaction model from the regression coefficients of the correct quadratic model, further clarifying the relationship between the two models. We present equations in this section for two reasons: 1) to elucidate the mathematical connection between quadratic and interaction models, which may be helpful to see algebraically, and 2) to provide a foundation for the empirical applications we present later. In particular, we use the equations shown below to transform reported statistics for an interaction model into plausible estimates for an (untested) quadratic model, all without using any of the original data.

Let us suppose that the true population model includes a quadratic effect of *X*, as shown in Eq. (7). Our goal is then to solve for the population values of the coefficients for the incorrect model shown in Eq. (8).

$$Y_i = \beta_0 + \beta_1 X_i + \beta_2 Z_i + \beta_3 X_i^2 + \epsilon_i; \tag{7}$$

$$Y_i = \beta_0^* + \beta_1^* X_i + \beta_2^* Z_i + \beta_3^* (X_i \times Z_i) + \epsilon_i^*. \tag{8}$$

The superscript * notation in Eq. (8) is used to differentiate the coefficients between the correct and incorrect models. To make it easier to solve for the coefficients in Eq. (8), we will assume that *Z* is a binary grouping variable and that *X* is normally distributed within groups.[6] These assumptions allow us to parallel derivations given in detail in Bauer (2005, appendix) to obtain the following formulas:

$$\beta_0^* = \beta_0 + \beta_3(\sigma_{X_0}^2 - \mu_{X_0}^2); \tag{9}$$

$$\beta_1^* = \beta_1 + 2\beta_3\mu_{X_0}; \tag{10}$$

$$\beta_2^* = \beta_3(\mu_{X_0}^2 - \mu_{X_1}^2 + \sigma_{X_1}^2 - \sigma_{X_0}^2); \tag{11}$$

$$\beta_3^* = 2\beta_3(\mu_{X_1} - \mu_{X_0}). \tag{12}$$

μ_{X_0} indicates the population mean of *X* given $Z = 0$ and μ_{X_1} is the mean of *X* given $Z = 1$. Similarly, $\sigma^2{}_{X_0}$ indicates the population variance of *X* given $Z = 0$ and $\sigma^2{}_{X_1}$ is the variance of *X* given $Z = 1$.

We are most concerned with the value of $\beta_3{}^*$, the regression coefficient for the spurious interaction effect. Based on the formula given in Eq. (12), we can see that $\beta_3{}^*$ will be a function of the true quadratic effect, β_3, and the difference between the means of *X* for the two groups (i.e., the correlation between *X* and *Z*). This indicates that $\beta_3{}^*$ is more pronounced when β_3 is large (i.e., there is greater curvature to the quadratic) and when the difference in the conditional group means of the focal predictor is substantial (i.e., the correlation between *X* and *Z* is stronger). Thus, these are the two primary factors that will drive the likelihood of detecting a spurious interaction effect when an incorrect interaction model is fit to sample data (in addition to sample size).

Aside from further explicating how quadratic effects can produce spurious interactions, these formulas offer the opportunity to translate results between models. That is, we can take the coefficients for an

[6] Extensions of our analytical derivations to more than two groups are trivial. In contrast, extensions to continuous *Z* variables or non-normally distributed predictors are more difficult, though the problem of interaction effects masquerading for quadratic effects remains present.

interaction model and invert these equations to infer what quadratic model might have yielded those results. Rearranging Eqs. (9)–(12) we obtain[7]

$$\beta_0 = \beta_3(\sigma^2_{X_0} - \mu^2_{X_0}) - \beta^*_0; \quad (13)$$

$$\beta_1 = 2\beta_3\mu_{X_0} - \beta^*_1; \quad (14)$$

$$\beta_2 = \mu_{Y_1} - \mu_{Y_0} - \beta_1(\mu_{X_1} - \mu_{X_0}) - \beta_3(\mu^2_{X_1} + \sigma^2_{X_1} - \mu^2_{X_0} - \sigma^2_{X_0}); \quad (15)$$

$$\beta_3 = \frac{\beta^*_3}{2(\mu_{X_1} - \mu_{X_0})}. \quad (16)$$

Here, μ_{Y_1} and μ_{Y_0} are the population means of Y for each group. Using these formulas, we can take a published example of an interaction model and answer the question, if this was really a quadratic effect masquerading as an interaction, what would the quadratic effect have looked like?

1.5. Empirical example

The example we consider comes from Pedrelli et al. (2011), who found that the effect of depression on alcohol consumption by college students differed between men and women. Pedrelli et al. fitted a two-way interaction model, where the outcome *Y* was alcohol use (specifically, the number of alcoholic drinks consumed per day), the focal predictor *X* was a depression score (mean-centered), and the moderator *Z* was gender (Female = 1 and Male = 0). The final model for the sample, in which all effects were statistically significant, was reported as:

$$Alcohol\ \ Use_i = 1.43 + .04Depression_i - .80Gender_i - .05(Depression \times Gender) + e_i, \quad (17)$$

where *e* are the sample residuals. The group means and standard deviations of depression were reported for males as $\overline{X}_{Male} = -1.13$ and $s_{Male} = 5.41$, and for females as $\overline{X}_{Female} = 0.95$ and $s_{Female} = 6.32$.[8] Additionally, the group means of alcohol use were reported as $\overline{Y}_{Male} = 1.43$ and $\overline{Y}_{Female} = 0.68$. Inserting these coefficient estimates and sample statistics into Eqs. (13)–(16) we determined that if the true model was actually quadratic, the coefficients would have been[9]

$$Alcohol\ \ Use_i = 1.77 + .01Depression_i - .65Gender_i - .01Depression^2 + e_i. \quad (18)$$

Fig. 5 compares the two models, replicating the simple slopes analyses published in Pedrelli et al. (2011) and overlaying the derived quadratic effect. In contrast to our earlier example in Fig. 4, here there is a non-zero effect for the grouping variable so we have two quadratic effect curves, identical except for being vertically offset from one another. Specifically, the curve for females is consistently 0.65 units lower than the curve for males.

The interpretation of the interaction effect in Pedrelli et al. was that male college students have a stronger relationship between depression and alcohol use than female college students. An alternative explanation, however, is that the interaction is spurious, resulting from an omitted quadratic effect of depression on alcohol use. The quadratic model that would have produced these results implies that drinking increases as depression moves from low to moderate levels but then decreases again at higher levels of depression. College students with

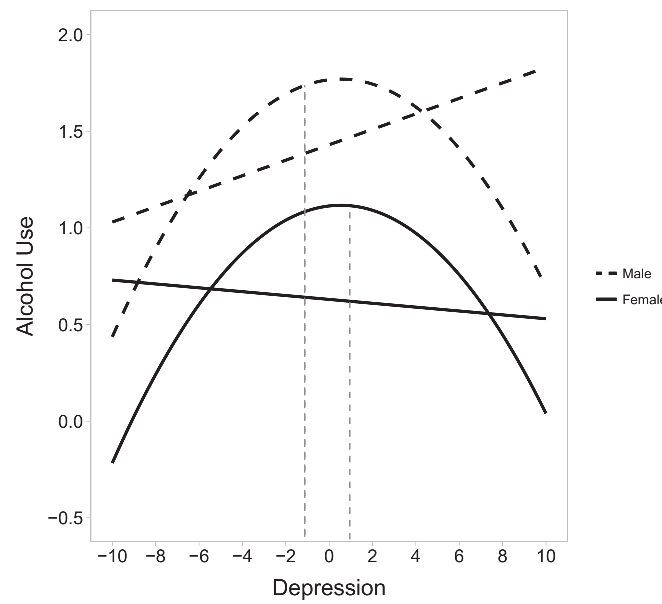


Fig. 5. Pedrelli et al. (2011) simple slopes analysis (straight lines) with overlay of derived alternative quadratic model (curved lines). Plotted functions may extend beyond the range of the observed data, as this was not reported.

near-average levels of depression drink the most. Rather than gender affecting the strength of this relationship between depression and alcohol use, gender is simply a covariate that shifts predicted alcohol use down for females and up for males.

Although the interaction effect reported by Pedrelli et al. *might* be this nonlinear effect in disguise that does not necessarily mean that it actually is. The interaction effect may well be more theoretically plausible (it looks that way to us). With access to the actual data, it might also be possible to empirically adjudicate between the two competing models (we discuss this possibility further in a later section). Nevertheless, we emphasize that without considering both possibilities – an interaction effect or a nonlinear relationship – researchers risk falsely interpreting one as the other. As we show next, this risk generalizes to more complex analyses as well.

2. Interaction effects and quadratic effects in moderated mediation

Consistent with much of the literature on this topic, to this point we have only considered the possibility of conflating interactions with nonlinear effects in the linear regression model. Here, we present novel results showing that this problem also manifests in models of greater complexity, focusing on moderated mediation analysis as a case in point. The literature on mediation, moderated mediation, and statistical inference for indirect effects is large, and given limited space, we refer readers to MacKinnon (2008) and Hayes (2017) for thorough treatments. Our goal here is simply to show how the validity of moderated mediation models is threatened by failure to consider possible nonlinear relationships between predictors and outcomes.

2.1. Moderated mediation

In a standard mediation analysis, the primary focus is how the effect of a predictor is transmitted to an outcome, with the goal of identifying intervening variables (mediators) that serve as mechanisms for this effect. The portion of the predictor's effect that passes through the intervening variables is called the indirect effect. If the indirect effect is non-zero, then the intervening variables at least partially mediate the effect of the initial predictor on the outcome. Moderated mediation

[7] Since β_2 is not found in Equations 9–12, we solved for its value using the other coefficients as well as additional descriptive statistics, namely the mean of the outcome in each group.

[8] Because Pedrelli et al. mean-centered depression before creating the product term, we report grand-mean-centered group means here.

[9] Under the assumption that depression is normally distributed within groups, which may or may not be realistic for these data.

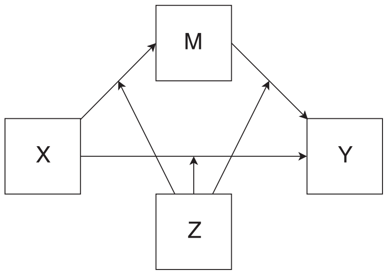


Fig. 6. Moderated mediation model in which M is a mediator of the effect of X on Y, and Z is a moderator of all effects in the model.

analysis is motivated by the idea that a given intervening variable may be a more important mediator for some people than others, such that the magnitude of the indirect effect varies across individuals. For instance, Madson, Moorer, Zeigler-Hill, Bonnell, and Villarosa (2013) found that protective behavioral strategies (e.g., having a designated driver) mediated the effect of positive expectations about alcohol use (e.g., social lubricant) on alcohol consumption for women but not men. This finding suggests that interventions aimed at modulating alcohol expectancies would be less effective for men than women.

Generally, moderated mediation can involve multiple mediators, multiple moderators, and the same moderator affecting multiple links in a mediation chain. An example of a moderated mediation model is shown in Fig. 6. Here, M mediates the effect of X on Y. Z can moderate either or both of the two components of the indirect effect, that is, the effect of X on M or the effect of M on Y. Z can also moderate the direct effect of X on Y. Not all of these moderation effects need be included in any given analysis. Whichever of these moderated effects are included, however, must be dissociated from possible nonlinear effects. As in the linear regression model, any omitted nonlinear effect could also masquerade as an interaction, potentially leading to false conclusions about moderated mediation effects.

To build up to the moderated mediation model we will begin by specifying a standard mediation analysis without moderation and then subsequently introduce the moderated effects. With one mediator M and one outcome Y, the model can be expressed as follows:

$$M_i = \alpha_0 + \alpha_1 X_i + \epsilon_{mi}; \tag{19}$$

$$Y_i = \beta_0 + \beta_1 X_i + \beta_2 M_i + \epsilon_{yi}. \tag{20}$$

The effect of X on M, α_1, captures the first part of the indirect effect, and the effect of M on Y, β_2, captures the second part. The product of these two coefficients, $\alpha_1\beta_2$, is the indirect effect. The direct effect of X on Y that does not pass through M is represented by β_1. The sum of the direct and indirect effects, $\beta_1 + \alpha_1\beta_2$, is the total effect of X on Y.

In moderated mediation analysis at least one of the effects in Eqs. (19) and (20) is hypothesized to vary as a function of a moderator variable Z. For instance, suppose that Z moderated the magnitude of only the M to Y component of the indirect effect (the $M \rightarrow Y$ path in Fig. 6). Then Eq. (20) would be extended as follows:

$$Y_i = \beta_0 + \beta_1 X_i + \beta_2 M_i + \beta_3 Z_i + \beta_4 (M_i \times Z_i) + \epsilon_{yi}; \tag{21}$$

$$= \beta_0 + \beta_1 X_i + \beta_3 Z_i + (\beta_2 + \beta_4 Z_i) M_i + \epsilon_{yi}; \tag{22}$$

where $\beta_2 + \beta_4 Z$ is the effect of M on Y. This interaction effect in turn implies that the indirect effect of M depends on Z (Edwards & Lambert, 2007; Preacher, Rucker, & Hayes, 2007):

$$\alpha_1(\beta_2 + \beta_4 Z) = \alpha_1\beta_2 + \alpha_1\beta_4 Z. \tag{23}$$

The first term, $\alpha_1\beta_2$, is the indirect effect when $Z = 0$ and the second term, $\alpha_1\beta_4 Z$, indicates the extent to which the indirect effect changes per unit change in Z. Similar to a simple slopes analysis, we can compute values for the indirect effect across selected levels of Z, called simple indirect effects, to gain a better understanding of the moderation effect. In this context, Hayes (2015) defined the coefficient $\alpha_1\beta_4$ as the *index of moderated mediation*, as it determines whether and how much the indirect effect changes with Z.

For simplicity, we have focused here only on moderation of the $M \rightarrow Y$ pathway, but it is also possible for Z to moderate the X to M pathway instead or to moderate both $X \rightarrow M$ and $M \rightarrow Y$ pathways. The formula for the simple indirect effects would change accordingly, but would be obtained similarly by multiplying the expressions for the two paths (either or both of which might be conditional on Z). As with the linear regression model, however, any one of the interaction effects within the model might actually represent a nonlinear effect in disguise and give the false appearance of moderated mediation.

2.2. Moderated mediation or quadratic mediation?

We showed before that an interaction effect in linear regression can serve as a proxy for a nonlinear effect. The same logic applies to mediation models where one or more indirect paths are thought to be moderated. For instance, it is clear that the interaction term, $M \times Z$, in Eq. (21) will share variance with the omitted quadratic term, $M \times M$, to the extent that M and Z are correlated. If the true functional form of the effect of M on Y (second part of the indirect effect) is actually nonlinear and not moderated by Z, then the model predicting Y would instead be:

$$Y_i = \beta_0 + \beta_1 X_i + \beta_2 M_i + \beta_3 Z_i + \beta_4 M_i^2 + \epsilon_{yi}. \tag{24}$$

This quadratic model in Eq. (24) implies that the indirect effect varies in magnitude over the range of M. To compute the indirect effect of this nonlinear model, partial derivatives may be taken of each pathway (Hayes & Preacher, 2010; Stolzenberg, 1980). In multivariable calculus, partial derivatives capture the rate of change in one variable (i.e., Y) with respect to a unit change in another (i.e., X), holding all others constant (i.e., Z). It is not a coincidence that this definition is synonymous to the interpretation of partial regression coefficients in multiple regression. Thus, taking the partial derivatives of M with respect to X (in Eq. (19)) and Y with respect to M ($\frac{\partial Y}{\partial M}$ in Eq. (24)) gives the nonlinear indirect effect as

$$\frac{\partial M}{\partial X}\frac{\partial Y}{\partial M} = \alpha_1(\beta_2 + 2\beta_4 M) = \alpha_1\beta_2 + 2\alpha_1\beta_4 M. \tag{25}$$

Comparing to the earlier indirect effect expression for the moderated mediation model in Eq. (23), we can see that the index of moderated mediation, $\alpha_1\beta_4$, multiplied by Z has been replaced by the *index of nonlinear mediation*, $2\alpha_1\beta_4$, multiplied by M.

Similar to the linear regression model, if a nonlinear effect of M on Y is omitted, and an M by Z interaction is included in the model by mistake, then a spurious interaction effect will be obtained to the extent that M and Z are correlated. In turn, this will lead to a spurious non-zero value for the index of moderated mediation. If we assume Z to be binary and M to be normally distributed within each level of Z then we can apply the same mathematical formulas given in the section on linear regression to solve for the precise magnitude of the spurious interaction and equally spurious index of moderated mediation.

Using Eq. (12), the spurious interaction is

$$\beta_4^* = 2\beta_4(\mu_{M_1} - \mu_{M_0}), \tag{26}$$

which produces a spurious index of moderated mediation:

$$\alpha_1\beta_4^* = 2\alpha_1\beta_4(\mu_{M_1} - \mu_{M_0}). \tag{27}$$

As can be seen, the spurious index of moderated mediation is a function of the magnitude of the quadratic effect, β_4, the strength of the relationship between M and Z (manifesting as the difference in the group means of M when Z is a grouping variable), and the magnitude of the $X \rightarrow M$ effect, α_1. As the strength of the quadratic effect increases and the relationship between M and Z rises (group means of M diverge to a greater degree), the index of moderated mediation becomes more pronounced in value. Because α_1 serves as a multiplier in this

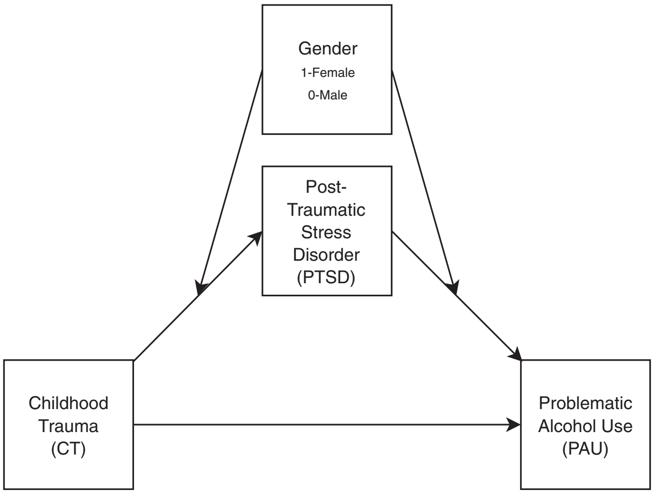


Fig. 7. Moderated mediation model specified in Cross et al. (2015).

expression, larger spurious values for the index of moderated mediation will also be obtained as α_1 increases.

As we now show, similar to the regression case, we can use derivations matching Eqs. (13)–(16) to take the results from a previously published moderated mediation analysis and determine what quadratic mediation model could have generated those results, if it was really the case that true quadratic effects were masquerading as false interactions.

2.3. Empirical example

We consider the example from Cross, Crow, Powers, and Bradley (2015) shown in Fig. 7. They found that post-traumatic stress disorder (PTSD) mediated the effect of childhood trauma (CT) on problematic alcohol use (PAU) in low-income, African-American men and women. Both the effect of CT on PTSD and the effect of PTSD on PAU were moderated by gender. The final model was reported as:

$$PTSD_i = -1.680 + .384CT_i + 1.399Gender_i - .065(CT_i \times Gender_i) + e_{mi}; \tag{28}$$

$$PAU_i = 5.736 + .038CT_i + .135PTSD_i - 3.79Gender_i - .049(PTSD_i \times Gender_i) + e_{yi}; \tag{29}$$

where e_m and e_y are the sample residuals for each model. The indirect effect was therefore equal to (.384 − .065*Gender*)(.135 − .049*Gender*) and the index of moderated mediation was −.024.[10] The group means and standard deviations of CT were $X_{Male} = 37.63$, $s_{Male} = 13.99$, $X_{Female} = 41.87$, and $s_{Female} = 19.03$. The means and standard deviations of PTSD were $M_{Male} = 12.77$, $s_{Male} = 12.27$, $M_{Female} = 13.08$, and $s_{Female} = 12.33$. Lastly, the means of PAU were $M_{Male} = 12.77$ and $M_{Female} = 13.08$.

Notably, because two paths of this model are moderated, there is the potential for either moderation effect to represent an omitted nonlinear effect. Replacing the population values in Eqs. (13)–(16) with the sample statistics and coefficient estimates, we can determine what the underlying quadratic equations might have looked like[11]:

$$PTSD_i = -11.034 + .961CT_i + .095Gender_i - .008CT_i^2 + e_{mi}; \tag{30}$$

$$PAU_i = 4.747 + 2.153PTSD_i - 4.148Gender_i - .079PTSD_i^2 + e_{yi}. \tag{31}$$

Figs. 8 and 9 compare the moderated mediation model equations, Eqs. (28) and (29), with the quadratic mediation model equations, Eqs. (30) and (31), respectively. In Fig. 8, the quadratic curves for males and females are indistinguishable because the effect of gender is so small. In Fig. 9, however, the male and female curves are separated by approximately 4 units.

Results from the original moderated mediation model indicated that the indirect effect was stronger for males than females. Alternatively, the interaction effects found by Cross et al. could have been wholly or partly spurious, resulting from omitted nonlinear effects of childhood trauma on PTSD and/or PTSD on problematic alcohol use. Fig. 8 suggests a plausible alternative nonlinear effect of childhood trauma on PTSD. The effect of childhood trauma is most acute at the low end of the scale, where even relatively small increases in childhood trauma are strongly predictive of increases in PTSD. At the higher end of the scale, however, differences in childhood trauma are no longer as predictive of differences in PTSD, which remains consistently high. This interpretation is consistent with a curve of diminishing return (where the slight downturn at the highest of CT would be interpreted as the result of model error, i.e., the quadratic being an imperfect approximation to an asymptotic curve). The moderation effect originally detected for the effect of CT on PTSD may thus have been a spurious result, a consequence of incorrectly omitting this nonlinear effect. By this explanation, the shallower slope found for women in the moderated mediation analysis only reflects their higher mean level of childhood trauma relative to men (i.e., the correlation between gender and CT).

Fig. 9 plots the other component of the indirect effect, namely the effect of PTSD on problematic alcohol use. As seen, given the relatively small mean difference in PTSD between men and women (low correlation between PTSD and gender), an extremely strong nonlinear effect would need to exist to produce the simple slope differences found in the moderation model. This nonlinear effect would require sharp increases in PAU as PTSD increases toward the mean followed by sharp decreases in PAU above the mean. Such a pattern does not seem plausible.

Based on these considerations we might conclude that the original results reported for the moderated mediation analysis are at most partly spurious. The most plausible alternative hypothesis is that the CT → PTSD relationship is actually nonlinear whereas the PTSD → PAU relationship is linear but varies by gender. Such a scenario would still drastically alter the interpretation of the results from the model.

Assuming the above scenario to be true, the nonlinear indirect effect would then be equal to (.961 + 2[−.008*CT*])(.135 + − .049*Gender*). A single index of moderated mediation cannot be computed here because the indirect effect is conditional on both a continuous trauma score and a binary gender variable. Nevertheless, the nonlinear indirect effect may be calculated across multiple values of both childhood trauma and gender and compared. For instance, at a standard deviation below the mean of CT, the difference between females and males on the indirect effect of CT on PAU through PTSD is −0.029; at the mean of CT, the difference between females and males on the indirect effect is −0.015; and at a standard deviation above the mean of CT, the difference is −0.001. From these results, the difference between females and males on the indirect effect of PTSD appears to vary across levels of childhood trauma. Fig. 10 illustrates how the indirect effect varies across CT between females and males for both the moderated mediation and nonlinear mediation model. In Fig. 10a, the indirect effect varies by gender only, corresponding to the indirect effect computed from Eqs. (28)–(29). Fig. 10b, in contrast, shows that the indirect effect varies not only by gender, but also by CT. The nonlinear indirect effect varies to a greater degree between men and women at smaller levels of CT, while at greater values of CT there appears to be little difference between genders (and no mediation of PTSD).

[10] The index of moderated mediation is computed differently than before. As Hayes (2013, pp. 411; 2015) explains, in the case of a common dichotomous (coded 0–1) variable moderating both indirect paths, the index of moderated mediation is a sum of coefficients: [(.384 × − .049) + (−.065 × .135) + (−.065 × − .049)].

[11] We assume normality of CT within men and women in the first equation and normality of PTSD within men and women in the second equation. These may or may not be reasonable assumptions.

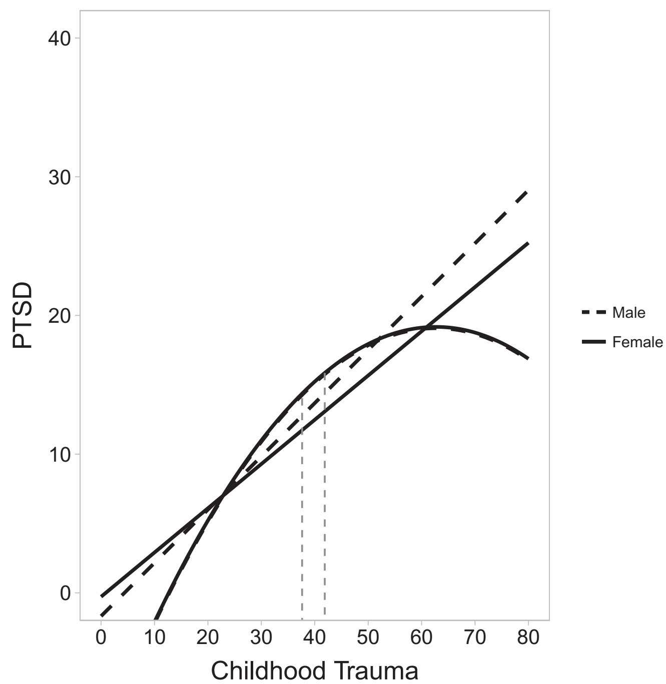


Fig. 8. Simple slopes analysis of the first indirect path in Cross et al. with overlay of the derived quadratic model. The gray vertical lines correspond to the group means.

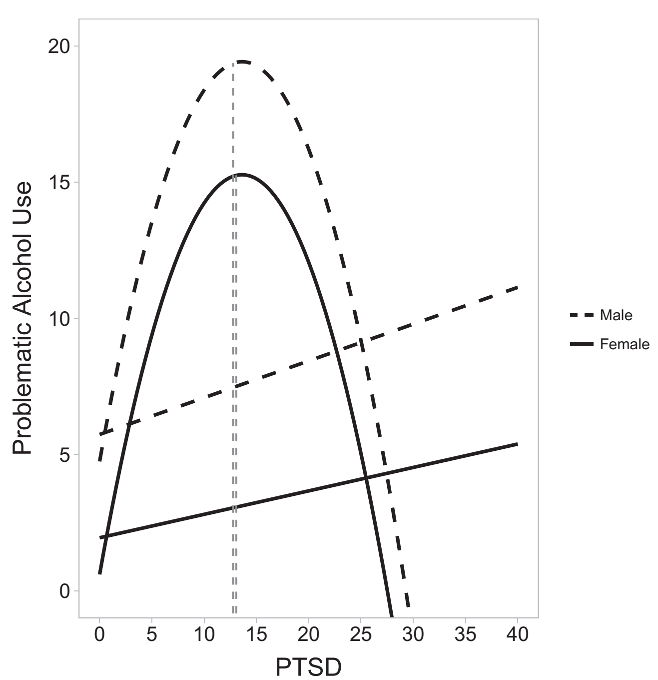
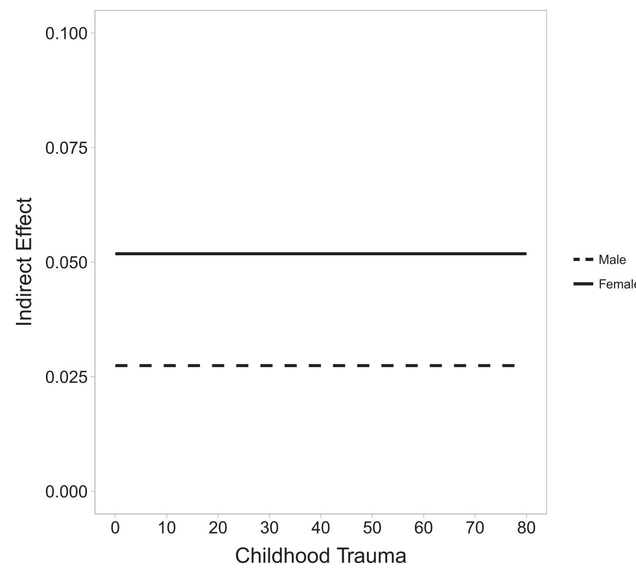
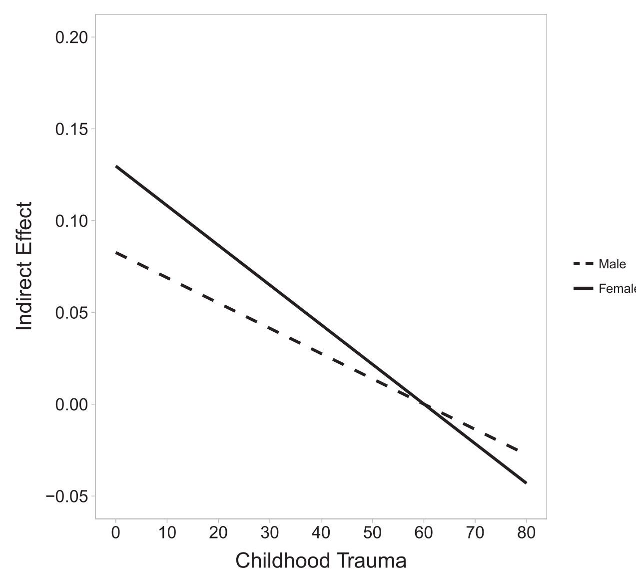


Fig. 9. Simple slopes analysis of second indirect path in Cross et al. (2015) with overlay of the derived quadratic model. The gray vertical lines correspond to the group means.

Substantively, the nonlinear indirect effect model, if true, would introduce an interesting nuance to the results: men and women experience different levels of PTSD and thus alcohol use in response to lower levels of childhood trauma only. At higher levels of trauma, the effect of trauma on alcohol use is not mediated by PTSD for either men or women. Of course, it is also possible that the model that Cross et al. (2015) originally presented, with moderated linear effects throughout, is indeed the correct one. The problem is that without consideration of the alternative nonlinear model



(a) Simple indirect effects for moderated mediation model.



(b) Simple indirect effects for nonlinear mediation model.

Fig. 10. Simple indirect effects plotted against childhood trauma. Computed from coefficients reported in Cross et al. (2015).

there is no way to know. It is thus incumbent upon the researcher to fully consider and attempt to adjudicate between these possibilities based on theory and empirical evidence. Deciding which model is superior could have serious implications. For instance, it is conceivable that clinical recommendations (e.g., treating PTSD given a particular level of childhood trauma) may depend on which interpretation is advocated by the researcher – moderated mediation or nonlinear mediation.

3. Recommendations

In this section, we review solutions that have been proposed for the problem of conflating quadratic effects for interactions. Most of these proposals have centered on the multiple regression model, but their extension to moderated mediation analysis or other more complex models is fairly straightforward (for moderated mediation there are just more equations to consider). After reviewing the proposed solutions we offer a series of practical recommendations.

Lubinski and Humphreys (1990) were the first to recommend an empirical approach to solving the problem. Their approach uses stepwise regression to identify which effect – interaction or quadratic – should be included in the model. First, the researcher estimates an additive regression model with the main effects only (e.g., X and Z). Then stepwise regression selects the higher-order effect (e.g., X^2, Z^2, or XZ) that explains the most criterion variance (R^2). MacCallum and Mar (1995) noted its equivalence to comparing R^2 from two separate regression models (comparing an interaction model with either quadratic models).

This approach has the advantage of being straightforward and easy to implement, but was challenged by MacCallum and Mar (1995), who argued that differential unreliability in the individual components of the product and squared terms could lead to an unfair comparison between the two types of effects. Because squared variables are typically less reliable than product variables (see Bohrnstedt & Marwell, 1978; Busemeyer & Jones, 1983), there is usually greater downward bias on quadratic effects than interaction effects, producing differential power to detect the two effects. In some situations, this difference is sufficiently great that a false interaction might still win out over a true quadratic effect, leading to the wrong result. In particular, MacCallum and Mar showed that the probability of selecting the wrong model is highest when: (a) the correlation between the component variables (e.g., X and Z) exceeds 0.7, especially when the correlation is as high as 0.9; (b) the reliabilities of the component variables are lower than 0.8; and (c) the sample size is below 300.

Since this method is not failsafe under all conditions, MacCallum and Mar (1995) suggested that researchers should also consider past empirical findings and theoretical plausibility when adjudicating between the two types of effects. They also proposed using latent variable models to solve the unreliability problem. As latent variables are free of measurement error, quadratic and interactive effects among latent variables can be compared directly without concern for differential attenuation due to unreliability. Unfortunately, at the time, methods for estimating structural equation models with interactive or quadratic effects were still at an early stage of development, and computational demands and estimation difficulties made it difficult for applied researchers to implement this recommendation. In the decades since, however, new methods for estimation have been developed, computers have become more efficient, and statistical software has simplified implementation, making MacCallum and Mar's recommendation far more practical in contemporary research (Kelava et al., 2011; Kelava, Moosbrugger, Dimitruk, & Schermelleh-Engel, 2008; Klein & Moosbrugger, 2000; Klein & Muthén, 2007; Marsh, Wen, & Hau, 2004).

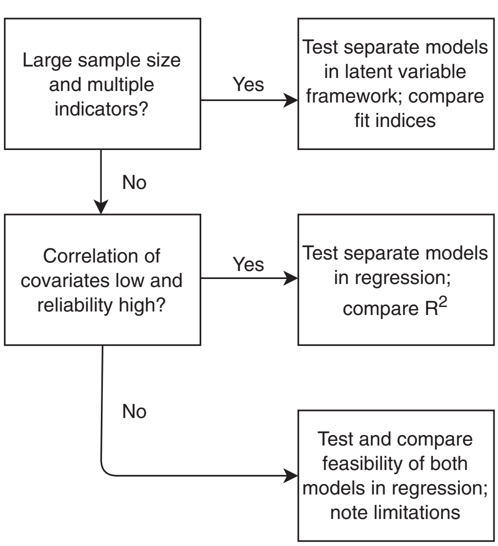


Fig. 11. Empirical procedure for distinguishing between nonlinear effects and interactions.

In light of these considerations, we suggest that researchers follow the recommendations outlined in Fig. 11 to address the problem of distinguishing interactive from nonlinear effects. Although we echo MacCallum and Mar's recommendation that researchers should consider prior empirical findings and develop theoretical support to possibly rule out nonlinear effects or interactions, we realize that theory and past research often do not provide clear guidance. For instance, Fig. 1 suggests that little empirical work has been done to distinguish interactions from nonlinear effects. We therefore recommend that researchers should try to empirically discriminate between models unless there is a strong theoretical reason not to. We recommend fitting competing models, one with a quadratic effect and the other with an interactive effect, to determine which model provides the best fit to the data.[12] Under unfavorable conditions, such as there being high multicollinearity and poor reliability of the predictors, this method will be inaccurate unless it is implemented in a latent variable framework. The latent variable approach, however, requires both a sufficient sample size and that the predictors be measured with multiple indicators (i.e., multi-item scales). When evaluating competing latent variable models, a variety of indices of fit can be used to compare empirical consistency with the data (e.g., RMSEA, Bayesian Information Criterion, etc.), analogous to comparing R^2 in competing non-nested regression models (Steiger, 1980). Even when data conditions are more favorable for model selection using observed variable models, we recommend using a latent variable modeling approach when possible to avoid bias in the effect estimates due to measurement error. It is worth noting, however, that the difference in fit between interaction and nonlinear models may sometimes be slight and both effects may be equally plausible. In these cases, researchers should consider both effects and call for future research that will better evaluate these competing hypotheses.

These recommendations also serve to guide future study design. When it is important to discriminate between nonlinear and interactive effects, researchers should invest in obtaining measures with high reliability, particularly when the predictors are likely to correlate highly. When attaining high reliability is not possible, researchers should plan for a larger sample size and be sure to include multiple indicator variables or items for each predictor to enable implementation of a latent variable modeling approach.

Although we have focused on recommendations within the multiple regression model, similar recommendations pertain for moderated mediation and nonlinear mediation. One caveat is that, given the greater complexity of the model, it may be more difficult to fit latent variable models including multiple nonlinear or interactive effects simultaneously. In that event, we recommend breaking the model up into separate models for (latent) M and Y and attempting to differentiate nonlinear from moderator effects for each path separately. In some cases, comparing the R^2 for each model in multiple regression may be more feasible, given good reliability and low multicollinearity between the predictors.

[12] In agreement with Aiken and West (1991) and MacCallum and Mar (1995), we do not recommend including all higher-order effects into a single model to identify the “winning” effect. Although Ganzach (1997) showed that excluding an interaction or quadratic effect when the true model included all such effects could lead to misleading results, we point out that small sample sizes are common in the behavioral sciences and that multicollinearity undoubtedly occurs between the interaction and quadratic terms (even after mean-centering), both of which lead to low power in detecting true effects. This issue is exacerbated when moved into a latent variable model. Despite these difficulties, however, recent methodological developments using Bayesian regularization techniques have made the inclusion of multiple latent variable interactions and nonlinear effects within the same model more tractable (Brandt, Cambria, & Kelava, 2018). Regularization methods are effective variable selection tools that may prove helpful in identifying competing interaction and nonlinear effects in tandem.

Notably, these recommendations assume that any nonlinear effects are well-approximated by a quadratic effect. Analogous strategies could be implemented to differentiate other nonlinear effects from interactions (e.g., log X, $\sqrt{X}$, etc.), although these may be more challenging, particularly in a latent variable model. Researchers may also want to test other higher-order effects, for instance X^2Z, which may be interpreted as whether the nonlinear effect of X changes as a function of Z. We recommend that more specific hypotheses should be driven primarily by theory; otherwise the number of tested effects will increase rapidly, and, as a result, Type I error may increase.

3.1. Concluding remarks

Our first goal in this article was to elaborate on prior research to show, mathematically and by example, how nonlinear effects – specifically, quadratic effects – can masquerade as interactions. Because so few nonlinear effects have been tested in the literature in the last 20 years, we believed a thorough reexamination of this thorny statistical issue was called for. Second, we extended this work to cover the more complex moderated mediation model. Moderated mediation has been increasingly used to test conditional processes in the behavioral and social sciences. We explicated how (quadratic) nonlinear mediation can masquerade as moderated mediation and that inferences can change as a result. This general issue affects other complex models as well. For example, Bauer (2005) demonstrated that failing to model nonlinear effects can compromise tests of measurement invariance across groups; Bauer and Curran (2003) showed that unmodeled nonlinear effects can lead to over-estimation of latent classes in mixture models; and Bauer and Cai (2009) demonstrated that neglected nonlinear effects can manifest as spurious random slopes and cross-level interactions in multilevel models. Finally, we provided concrete recommendations for researchers based on work from Lubinski and Humphreys (1990) and MacCallum and Mar (1995), but updated to reflect more recent progress on fitting nonlinear latent variable models. Fig. 11 offers a blueprint for researchers to empirically distinguish interactive from nonlinear effects in both linear regression and moderated mediation.

We hope that by further explicating and demonstrating this problem, this paper will help researchers become better attuned to the need to evaluate the linearity of predictor effects in their models and the potential consequences of failing to do so, especially when evaluating moderation hypotheses. In many cases, it is possible to discriminate theoretically and empirically between nonlinear effects and interaction effects. Doing so should become a matter of routine practice.

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Contributors

DJ Bauer and WCM Belzak designed the study. WCM Belzak wrote the first draft of the manuscript. DJ Bauer provided technical expertise. All authors contributed to and have approved the final manuscript.

Conflicts of interest

None.

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