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I want to disaggregate the within- and between-effects of the predictors in my model. Should I use group-mean centering on X and/or M to accomplish this?

Note: In the response below I will refer to groups as the clusters, e.g., students within schools, but the same comments would apply to repeated measures within persons… group-mean centering would simply become person-mean centering and within- and between-group effects would become within- and between-person effects.

We did not directly address the centering / disaggregation issue in our paper but it has since come up often in emails and conversations. One way to do this would be to estimate a full multilevel SEM (e.g., in Mplus) rather than trick the MLM software into fitting the model. The multilevel SEM approach would allow one to estimate a between-groups mediation model and a within-groups mediation model simultaneously, the latter potentially including random intercepts and slopes.

I believe, however, that the approach we originally recommended can also be used to do this, with some minor modifications. The original model was given as

$$\begin{aligned} M_{ij} &= d_{Mj} + a_j X_{ij} + e_{Mij} \\ Y_{ij} &= d_{Yj} + b_j M_{ij} + c'_j X_{ij} + e_{Yij} \end{aligned}$$

where the random effects follow a multivariate normal distribution:

$$\begin{pmatrix} d_{Mj} \\ a_j \\ d_{Yj} \\ b_j \\ c'_j \end{pmatrix} \sim N\left[\begin{pmatrix} d_M \\ a \\ d_Y \\ b \\ c' \end{pmatrix}, \begin{pmatrix} \sigma^2_{d_{Mj}} & & & & \\ \sigma_{d_{Mj},a_j} & \sigma^2_{a_j} & & & \\ \sigma_{d_{Mj},d_{Yj}} & \sigma_{a_j,d_{Yj}} & \sigma^2_{d_{Yj}} & & \\ \sigma_{d_{Mj},b_j} & \sigma_{a_j,b_j} & \sigma_{d_{Yj},b_j} & \sigma^2_{b_j} & \\ \sigma_{d_{Mj},c'_j} & \sigma_{a_j,c'_j} & \sigma_{d_{Yj},c'_j} & \sigma_{b_j,c'_j} & \sigma^2_{c'_j} \end{pmatrix}\right]$$

This model is then re-expressed by stacking the Y and M on top of one another to make Z, creating the indicator variables S_M and S_Y (scored 1 when Z represents M or Y, respectively) and fitting the model via the single equation here:

$$\begin{aligned} Z_{ij} = S_{M_{ij}}&\left(d_{Mj} + a_j X_{ij}\right) + \\ &S_{Y_{ij}}\left(d_{Yj} + b_j M_{ij} + c'_j X_{ij}\right) + e_{Zij} \end{aligned}$$

with the error term permitted to be heteroscedastic across values of Z that represent M versus Y.

The modified model that separates within- and between-group effects would be written instead as

$$\begin{aligned} M_{ij} &= d_{Mj} + a_{wj}\dot{X}_{ij} + a_b\bar{X}_{\cdot j} + e_{Mij} \\ Y_{ij} &= d_{Yj} + b_{wj}\dot{M}_{ij} + c'_{wj}\dot{X}_{ij} + b_b\bar{M}_{\cdot j} + c'_b\bar{X}_{\cdot j} + e_{Yij} \end{aligned}$$

where the b and w subscripting indicates a between-cluster effect versus a within-cluster effect; $\dot{X}$ and $\dot{M}$ are cluster-mean-centered predictors and $\bar{X}$ and $\bar{M}$ are cluster means.

The distribution of the random effects is now given as

$$\begin{pmatrix} d_{Mj} \\ a_{wj} \\ d_{Yj} \\ b_{wj} \\ c'_{wj} \end{pmatrix} \sim N\left[\begin{pmatrix} d_M \\ a_w \\ d_Y \\ b_w \\ c'_w \end{pmatrix}, \begin{pmatrix} \sigma^2_{d_{Mj}} & & & & \\ \sigma_{d_{Mj},a_{wj}} & \sigma^2_{a_j} & & & \\ 0 & \sigma_{a_{wj},d_{Yj}} & \sigma^2_{d_{Yj}} & & \\ \sigma_{d_{Mj},b_{wj}} & \sigma_{a_{wj},b_{wj}} & \sigma_{d_{Yj},b_{wj}} & \sigma^2_{b_{wj}} & \\ \sigma_{d_{Mj},c'_{wj}} & \sigma_{a_{wj},c'_{wj}} & \sigma_{d_{Yj},c'_{wj}} & \sigma_{b_{wj},c'_{wj}} & \sigma^2_{c'_{wj}} \end{pmatrix}\right]$$

Note that relative to the first specification, one element of the random effects covariance matrix, the covariance between the random intercepts, $\sigma_{d_{Mj},d_{Yj}}$, has been set to zero. This is because the between-groups relationship between M and Y is now captured by the regression slope b_b. Mediation and indirect effects are evaluated at the within level using the formulae in the original paper; mediation at the between level can be evaluated using the more traditional $a_b b_b$ product.

This model can similarly be re-expressed as

$$Z_{ij} = S_{M_{ij}}\left(d_{Mj} + a_{wj}\dot{X}_{ij} + a_b\bar{X}_{\cdot j}\right) + \\ S_{Y_{ij}}\left(d_{Yj} + b_{wj}\dot{M}_{ij} + c'_{wj}\dot{X}_{ij} + b_b\bar{M}_{\cdot j} + c'_b\bar{X}_{\cdot j}\right) + e_{Zij}$$

Where (uncentered) M is again stacked on Y to make Z. Residuals are again heteroscedastic. The model can be fit in SAS using the syntax given here (*xc* and *mc* are cluster-mean-centered versions of X and M and *mx* and *mm* are the cluster means of X and M):

```
proc mixed data=BWlong asycov noclprint method=reml maxiter=1000;
class dep;
model z = Sm Sm*xc Sm*mx
          Sy Sy*mc Sy*mm Sy*xc Sy*mx /noint solution covb ddfm=bw;
random Sm Sy Sm*xc Sy*mc Sy*xc/g gcorr type=un subject=id;
repeated / subject=id r group=dep;
parms (.6)
      (0) (.4)
      (0) (0) (.160)
      (0) (0) (.113) (.160)
      (0) (0) (0) (0) (.040)
      (.65) (.45) / hold=2;
run;
```

With the random effects ordered as shown in the RANDOM statement, the second element in the PARMS statement should always be zero. The other elements are start values and can be determined either by educated guess or by fitting the two univariate models first (one for M and one for Y) to get empirical estimates for most of the values. Current research is investigating the performance of this approach as a means of decomposing within- and between-cluster mediation effects with random slopes.

Advantages of this approach, relative to multilevel SEM, are access to complex error structures in standard MLM programs and small sample estimation (i.e., REML) and inference (e.g., t-tests on fixed effects rather than asymptotic z-tests). SEM programs can sometimes implement complex error structures but usually don't have the small-sample features of standard MLM programs. Also, to include the random slopes in a multilevel SEM requires that you do numerical integration or perhaps switch to a Bayesian estimator. So estimation becomes much more complex relative to the standard MLM.