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Conducting Tetrad Tests of Model Fit and Contrasts of Tetrad-Nested Models: A New SAS Macro

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This article describes a SAS macro to assess model fit of structural equation models by employing a test of the model-implied vanishing tetrads. Use of this test has been limited in the past, in part due to the lack of software that fully automates the test in a user-friendly way. The current SAS macro provides a straightforward method for researchers to use the vanishing tetrads implied by models to assess the fit of (a) structural equation models containing continuous endogenous variables; (b) structural equation models containing continuous endogenous variables nested for vanishing tetrads; and (c) structural equation models containing dichotomous, ordinal, or censored endogenous variables. Besides providing an alternative assessment of model fit to the usual likelihood-ratio test (LRT), the vanishing tetrads test occasionally provides a statistical assessment of competing models nested for vanishing tetrads but not nested for the LRT. The macro permits formal comparisons between tetrad-nested structural equation models containing dichotomous, ordinal, or censored endogenous variables.

A key focus of structural equation modeling (SEM) is the assessment of model fit. The usual test applied for assessing model fit is the likelihood-ratio chi-square test

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(LRT), which formally contrasts the lack of correspondence between the model-implied and sample covariance matrices (and, in some instances, means). Dissatisfaction with various features of this test has led to the development of a bevy of alternative fit indexes; however, nearly all are based on the concept of comparing the model-implied covariance matrix with the sample covariance matrix. Although clearly useful, other avenues of assessing model fit may yield important additional information, and provide tests that might otherwise not be possible. In particular, as first suggested by Bollen (1990) and later elaborated by Bollen and Ting (1993), a simultaneous test of the model-implied *vanishing tetrads* may provide a useful alternative to more traditional measures of model fit.

The vanishing tetrads test has several favorable properties. First, unlike many fit indexes, the vanishing tetrads test provides a formal assessment of model fit that can be compared with the usual LRT. Second, some models not identified in the usual sense can nevertheless be evaluated by the vanishing tetrads test. Similarly, some models that are not nested in the usual LRT sense are tetrad-nested, allowing the researcher to perform formal statistical comparisons of these models rather than resorting to heuristic assessments (e.g., minimum Bayes' Information Criterion). Third, certain nested models involve constraining one or more parameters to a boundary value, violating the regularity conditions of the conventional LRT (e.g., testing the dimensionality of latent variables by constraining their correlation to one). These nested models can nonetheless be tested for vanishing tetrads. Fourth, the vanishing tetrad test can be applied to polychoric and polyserial correlation (covariance) matrices for dichotomous, ordinal, or censored endogenous variables (Hipp & Bollen, 2003). This is an important advantage given the considerable uncertainty surrounding model fit assessment in structural equation models with noncontinuous outcomes. Related to this point, even when two models are nested in their parameters, some current strategies for assessing model fit for data with dichotomous, ordinal, or censored endogenous variables (e.g., diagonally weighted least-squares with mean and variance adjusted chi-square) do not allow formal nested model comparisons; again, the vanishing tetrads test provides a formal test between such models.

Despite these numerous advantages of the vanishing tetrads test, it has to date been utilized infrequently in empirical work. We speculate that there are four primary reasons for this state of affairs. One is that, until recently, the vanishing tetrads test was only applicable for models including all continuous endogenous variables. However, as mentioned, the recent work of Hipp and Bollen (2003) has extended this test to noncontinuous endogenous variables. A second reason for the lack of implementation of this test is more conceptual. Specifically, it is difficult to conceptualize a tetrad (unlike, say a correlation or covariance), or to identify on the basis of a path diagram which tetrads are or are not vanishing. Tedious covariance algebra is often required to identify the nonredundant vanishing tetrads implied by a model. Similarly, it is often nonobvious which models are tetrad-nested. For instance, Hipp and Bollen showed that a latent curve model and an autoregressive change model are tetrad-nested, a result one would likely not arrive at by intuition or inspection of path diagrams alone. A third possible reason for lack of use of the test is ambiguity over which tetrads to test. As we describe shortly, many different combinations of vanishing tetrads may be equally valid, leading to ambiguity and uncertainty over which set of tetrads to test or whether the results are sensitive to the particular set selected. Finally, we also believe that a key reason the vanishing tetrad test has not been widely implemented is that, until now, there has been no practical way for applied researchers to routinely employ such tests because of the lack of user-friendly software. Ting (1995) provided an SAS macro that allowed the researcher to test the vanishing tetrads of models when the observed data were continuous, but still required considerable analytical skill on the part of the researcher and did not provide for tests of models with noncontinuous outcomes or tests of nested models.

The SAS macro we describe here is user-friendly in several ways and also includes many of the recent important extensions of the vanishing tetrads test. First, the current macro allows the researcher to assess the vanishing tetrads of the model when the data come from dichotomous, ordinal, or censored observed variables. In such a case, the researcher need only provide the polychoric correlation (covariance) matrix and its accompanying asymptotic covariance matrix (ACM). Second, the macro automates many of the more onerous and analytically difficult tasks required to perform the vanishing tetrad test. For instance, the procedure of determining the vanishing tetrads implied by the model (done manually in the Ting [1995] macro) has now been fully automated, a considerable advantage, particularly for larger models. In addition, given two models, the program automatically detects whether the models are tetrad-nested and selects the appropriate vanishing tetrads for the test between models.[1] Third, Hipp and Bollen (2003) suggested that because there is generally not a unique set of vanishing tetrads for any given model the researcher may wish to pull random draws of vanishing tetrads to assess the sensitivity of the results to such selection. This randomization procedure is a fully automated feature of the current macro. Last, this macro has adopted a new strategy for selecting the vanishing tetrads implied by the model using the sweep operator. Hipp and Bollen pointed out that this strategy is about 60 times faster than the procedure suggested by Bollen and Ting (1993), a feature that becomes particularly important in randomizing sets of tetrads, bootstrapping applications, or conducting Monte Carlo studies of the test.

As a result of these improvements, this program provides a viable alternative to the conventional LRT for applied researchers wishing to test the vanishing tetrads

[1]The researcher must be certain that the vanishing tetrads of one model are a subset of the vanishing tetrads of the larger model, a nonobvious task. This necessity constituted a considerable challenge to the researcher attempting to make this determination manually.

of a model, regardless of whether the data are continuous or come from dichotomous, ordinal, or censored variables. This is particularly important as it provides an easy way for researchers to test nested models when the data contain dichotomous, ordinal, or censored variables.

We now outline the analytical basis and practical implementation of this new program. The next section briefly outlines the procedure for testing vanishing tetrads. After that, we explicate the vanishing tetrads test for dichotomous, ordinal, or censored data. We then introduce and describe how to implement the CTANEST1 SAS macro for testing vanishing tetrads. This is followed by an empirical example illustrating the use of the program, after which the article concludes.

PROCEDURE FOR TESTING VANISHING TETRADS

In this section we briefly describe the procedure involved in testing the vanishing tetrads implied by a model. Although previous researchers employed the concept of vanishing tetrads in an exploratory manner for discovering possible models (Glymour, Scheines, Spirtes, & Kelly, 1987), Bollen (1990) first proposed that model fit could be assessed by simultaneously testing the multiple vanishing tetrads implied by a model. This was later elaborated by Bollen and Ting (1993) to show that it not only could be used to assess the fit of structural equation models, but that in some instances it could be used to assess the fit of models that are not formally identified. A tetrad is formed from four random variables, and refers to the difference between the product of one pair of covariances and the product of the other pair. Four variables contain six covariances, and from these we can create three tetrads:

$$\tau_{1234} = \sigma_{12}\sigma_{34} - \sigma_{13}\sigma_{24} \,,$$
$$\tau_{1342} = \sigma_{13}\sigma_{42} - \sigma_{14}\sigma_{32} \,, \tag{1}$$

and

$$\tau_{1423} = \sigma_{14}\sigma_{23} - \sigma_{12}\sigma_{43} \,,$$

This notation comes from Kelley (1928), with τ_{ghij} referring to $\sigma_{gh}\sigma_{ij}$ - $\sigma_{gi}\,\sigma_{hj}$ and σ as the population covariance of the two variables that are indexed below it. A hypothesized model structure will imply that for some of these tetrads, $\tau_{ghij} = 0$, and these are referred to as vanishing tetrads. Given the set of implied vanishing tetrads in a model, Bollen (1990) proposed a method to simultaneously test whether this set of tetrads is significantly different than zero. Rejecting this hypothesis would suggest a possible problem with the hypothesized model. Failure to reject indicates consistency between the model and the data.

Given a theoretically specified model, the vanishing tetrad testing procedure has three steps: (a) identify all of the model-implied vanishing tetrads, (b) select an independent set of vanishing tetrads, and (c) form the simultaneous test statistic for the independent vanishing tetrads.

One method for determining the model-implied vanishing tetrads is through covariance algebra, but this is a tedious, error-prone procedure. We thus follow the empirical approach of Bollen and Ting (1993), which requires only that the researcher specify the hypothesized model and obtain its model-implied covariance matrix. From this matrix, the CTANEST1 macro can determine which tetrads are within rounding error of zero, or vanishing.

To find an independent set of vanishing tetrads, we use the sweep operator with the asymptotic covariance matrix (Σ_{tt}) for the set of vanishing tetrads in the model (Hipp & Bollen, 2003). If we had the population Σ_{tt} matrix there would be exact linear dependence between nonindependent tetrads, and the sweep operator would return values of zero for the redundant tetrads. However, because of floating-point errors and rounding due to imprecision of estimation in the model-implied Σ_{tt}, the sweep operator must find tetrads that are approximately linearly dependent on the others. By using a suitable criterion value for assessing linear dependence, the redundant tetrads can be removed to yield a smaller set of independent vanishing tetrads.[2] Note that for any model there may be several possible combinations of vanishing tetrads that will satisfy this criterion. To assess the sensitivity of the test results to the particular set selected, the CTANEST1 macro allows randomized selection of multiple sets of nonredundant vanishing tetrads.

Finally, the CTANEST1 macro forms the simultaneous test statistic for the set of independent vanishing tetrads selected and provides this result as output. In the case of continuous data, the researcher provides the sample covariance or correlation matrix; Bollen (1990) derived a modification of the test for the tetrad differences of correlation coefficients rather than covariances. In the case of noncontinuous data, the researcher provides the polychoric matrix and its accompanying weight matrix, as we describe next.

VANISHING TETRADS WITH NONCONTINUOUS DATA

Although applied researchers using structural equation models often are confronted with noncontinuous endogenous observed variables, assessing model fit in such instances is still a topic of considerable recent research and uncertainty. Because using the sample covariance matrix is not appropriate (as it fails to take into account the categorical nature of the data), a common strategy is to compute and employ a polychoric correlation (covariance) matrix by assuming that underlying the categorical observed variables are continuous variables (Jöreskog & Sörbom,

[2]Because these tetrads are only approximately linearly dependent, we must use a criterion value when determining those that are close to linearly dependent. Often, a value of 1E-8 is given as an appropriate value for determining linear dependence when using the sweep operator (Goodnight, 1978). We have found that in practice such a value works fairly well. Additionally, inputting a model-implied covariance matrix with greater degrees of precision also helps when using this procedure.

1984; Muthén, 1984; Olsson, 1979). Once this polychoric matrix and its accompanying asymptotic covariance matrix are obtained, there are then competing strategies for estimating the model, including maximum likelihood (Jöreskog, Sörbom, du Toit, & du Toit, 1999), weighted least-squares (Browne, 1984), and diagonally weighted least-squares (DWLS) (Jöreskog & Sörbom, 1984; Jöreskog et al., 1999; Muthén, 1993; Muthén & Muthén, 2001). However, with these approaches, the chi-square test of overall model fit often requires adjustments and sometimes does not perform well with the sample sizes characteristic of most applied research. Moreover, these adjustments often preclude formal nested-model comparisons.[3]

To address this issue, Hipp and Bollen (2003) extended the vanishing tetrads test to include observed variables that are dichotomous, ordinal, or censored. The test is modified by performing it using the polychoric correlation matrix and its asymptotic weight matrix (which need not be inverted). The ACM along with the polychoric matrix provides information on the unobserved underlying continuous variables. As in the case of continuous data, the tetrad test allows for tests of fit of some models that are not identified in the normal fashion. Additionally, the ability to test models nested for vanishing tetrads overcomes a particular limitation of the DWLS approach to model fit assessment in structural equation models when using noncontinuous data.

STEPS FOR USING CTANEST1

In this section we illustrate the use of CTANEST1. CTANEST1 is a macro for SAS that can be used in one of three ways: (a) copy the macro into a SAS program; (b) store the macro as a member of the SAS autocall library and use the SASAUTOS system option to call it; or (c) save the macro as a file and use the *%include* statement to incorporate the file into an SAS program. We describe the latter option here. When testing models with continuous data, this SAS macro can be used in combination with any SEM software program. When testing models with dichotomous, ordinal, or censored variables, this SAS macro can be used in combination with any SEM software program that estimates models for categorical data by calculating a polychoric correlation (covariance) matrix and its accompanying weight matrix. Programs such as LISREL and M*plus* currently are capable of estimating such models, and others will no doubt soon have such capabilities.[4] There is an accompanying CTAFILE macro that can also be used if the researcher wishes to bring in certain matrices as files; use of this is also described.

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[3]Some recent work by Satorra (2000) has suggested a method for computing the chi-square for nested tests on continuous nonnormal data when using the Satorra–Bentler corrected chi-square.

[4]The software program and complete documentation can be downloaded from this Web site: http://www.unc.edu/~johnhipp/ctanest1.htm

To perform a tetrad test of models, the researcher will need to provide (a) the sample correlation or covariance matrix (for continuous data) or the polychoric correlation (or covariance) matrix (for noncontinuous data); (b) the model-implied covariance matrix for each of the models that are to be tested; and (c) only for noncontinuous data, the accompanying asymptotic covariance matrix.

1. Estimating the First SEM Model and Obtaining the Sample Covariance or Polychoric Matrix

The researcher specifies the first model (or the only model, if not performing a nested test) in a SEM software program. If the data are ordinal, dichotomous, or censored, the researcher will want to obtain the polychoric correlation matrix; otherwise the sample covariance matrix will be obtained.[5] These matrices are easily obtained from many different SEM software programs, and can either be obtained directly from the output by cutting and pasting them into SAS when using the CTANEST1 macro, or output as files to be read into the CTAFILE macro.

2. Obtaining the Model-Implied Covariance Matrices

In this step the researcher specifies the model on which the tetrad test will be performed. In addition to specifying the model, the researcher must also request that the model-implied covariance matrix be output, which can be accomplished in most SEM software programs. If the researcher is testing nested models, then each model must be specified and its resulting model-implied covariance matrix retrieved.

3. Obtaining the Asymptotic Covariance Matrix

When using dichotomous, ordinal, or censored variables, the researcher will need to obtain the asymptotic covariance matrix (ACM). This also can be done in the SEM software program being used.[6] These matrices can either be obtained directly

[5]As of this writing, current software is unable to handle missing data when computing the polychoric matrix. As a result, listwise deletion is the default procedure employed by software programs. An alternative is for the researcher to adopt a multiple imputation strategy (Rubin, 1987; Schafer, 1997). Bringing these multiply imputed data sets into the software program of choice, the procedures described here would be followed in the same manner multiple times (for each imputed data set). These results could be combined into a single model fit index following the technique outlined by Meng and Rubin (1992).

[6]For instance, in M*plus* simply specify "residual" on the options line. In LISREL, including the RS subcommand on the output line will result in the "fitted covariance matrix" being printed in the output. When using AMOS, simply click on View-Analysis Properties, then on Output in the box that appears, then on Implied Moments. The table output will then provide a matrix of the model-implied covariance matrix with the nice feature of being able to adjust the precision of the values in the output. This additional precision is helpful for the program in attempting to determine the number of vanishing tetrads in the model.

from the output by cutting and pasting them into SAS when using the CTANEST1 macro, or they can be output as files to be read into the CTAFILE macro.

4. Using CTANEST1 to Perform the Nested Test

4a. Reading in the external files. The researcher will next need to call the CTANEST1 macro in SAS (and the CTAFILE macro if bringing in an ACM for categorical data). If using categorical data with a polychoric matrix, declare the CTAFILE macro in the following manner:

```
%include 'c:\tetrad\ctanest\ctafile.mac';
%ctafile(vars=7, mplus=1, pcm=1,
pcmfile=c:\tetrad\ctanest\wuth2.pcm,
acmfile=c:\tetrad\ctanest\wuth2.acm);
```

Note that the "%include" line tells SAS where the macro program is located (including the path). The following options are available:

vars—Declares the number of observed variables in the model.

mplus—Tells the macro which program was used to estimate the polychoric matrix. mplus = 1 declares it from M*plus,* otherwise mplus = 0 is the default.

lisrel—Tells the macro which program was used to estimate the polychoric matrix. lisrel = 1 declares it from LISREL, otherwise lisrel = 0 is the default.

pcm—Tells the macro if a polychoric matrix is being brought in as a file. pcm = 1 declares that a file is being brought in, otherwise pcm = 0 is the default.

pcmfile—This declares the folder and name of the polychoric matrix file (if one is being brought in).

acmfile—This declares the folder and name of the asymptotic correlation matrix file (if one is being brought in).

Note that these files are only saved in active memory (and are not written to a permanent location). Therefore, this program must be run during the same SAS session as when running the CTANEST1 macro.

4b. Performing the tetrad test with CTANEST1.mac (either categorical or continuous data). The macro is declared in the following manner:

```
%include 'c:\tetrad\ctanest\ctanest1.mac';
%ctanest1(SAMPMAT1 =
1
0.71 1
0.552 0.656 1
0.464 0.457 0.511 1
0.459 0.376 0.475 0.758 1
```

```
0.32 0.382 0.473 0.505 0.531 1
0.386 0.406 0.312 0.266 0.404 0.165 1,
IMPMAT1B =
0.51479341
0.1928794 0.49057623
0.27620327 0.28052742 0.91994148
0.31287078 0.31776899 0.45504514 1.01303695
0.25483948 0.25882916 0.37064332 0.41984828 0.73014457
0.22667554 0.2302243 0.32968117 0.37344817 0.30418096 0.95060927
0.11762292 0.11946439 0.17107299 0.19378388 0.15784082 0.14039683
0.51648281,
IMPMAT2B =
0.51426886
0.26890206 0.48999186
0.3228337 0.34913667 0.9181711
0.25950637 0.28064973 0.3369375 1.01242516
0.21699421 0.23467387 0.28174062 0.55646871 0.73400267
0.16513136 0.17858548 0.21440301 0.42346952 0.35409702 0.95137942
0.13404366 0.1449649 0.17403941 0.18368124 0.1535907 0.11688165
0.51648949,
N = 227, vars = 7, nesttest = 1, pchor = 0 , lisrel = 0, mplus = 1,
lowdiag =1, reps = 1);
run;
```

The options to be declared are each separated by a comma:

SAMPMAT—The sample covariance matrix (or polychoric matrix) is entered here, unless it is being brought in as a file from CTAFILE (with spaces between entries; note that it can simply be entered as a vector and CTANEST1 will transform it into a matrix with proper dimensions).

IMPMAT1B—The model-implied covariance matrix for the first model (the one with more vanishing tetrads) is entered here. Or, if testing just a single model, the implied covariance matrix should be entered here.[7]

IMPMAT2B—The model-implied covariance matrix for the second model (the one with fewer vanishing tetrads) is entered here.

N—This value reflects the sample size being tested.

vars—This value reflects the number of observed variables in the test.

nesttest—This should be set to 1 if the researcher wants to compare two nested models, while the default is 0 for testing just one model.

pchor—This should be set to 1 if the researcher is bringing in a polychoric correlation matrix as a file, otherwise 0 is the default.

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[7]Note that if the researcher inadvertently reverses the order of the model-implied matrices for the two models in a nested test, the program will correct this error and output a notification of this. When using the nested test option, the estimated chi-square for the model with fewer vanishing tetrads and the estimated chi-square for the difference in the two chi-squares are accurate. However, to estimate the chi-square for the model with more vanishing tetrads, the researcher should specify this model in a non-nested test.

lisrel—This should be set to 1 if the researcher has categorical data and is bringing in the ACM from LISREL, otherwise the default is 0.

mplus—This should be set to 1 if the researcher has categorical data and is bringing in the ACM from M*plus,* otherwise the default is 0.[8]

lowdiag—This should be set to 1 if the sample covariance (or polychoric) matrix and model-implied covariance matrices are lower diagonal (the program will symmetrize them), otherwise 0 is the default.

reps—This represents the number of randomization replications requested from the program. The default is 1 (to get a single test).

EXAMPLE: COMPARING TWO NESTED MODELS

Rating the Characteristics of a Good Citizen

We next briefly illustrate the use of the CTANEST1 macro with an empirical example. The example is a simple confirmatory factor analysis problem using categorical data. In this example we address the question of the appropriate number of dimensions to the concept under question: how individuals define being a good citizen. We will contrast one- and two-factor models. There are two reasons to prefer the confirmatory tetrad test for testing the dimensionality of this scale to an LRT. First, to test between one- and two-factor models, the two-factor model is specified and the correlation is either freely estimated or constrained to unity (implying only one factor is present). This constraint is on the boundary of the parameter space, violating one of the regularity conditions of the usual LRT. The vanishing tetrads test does not suffer from the same problem. A second reason for preferring the tetrad test in this instance is that it provides a natural test of nested models for categorical data using the polychoric correlation matrix. In contrast, the DWLS approach with a mean and variance corrected chi-square requires estimates of the degrees of freedom of each model that make nested chi-square tests difficult to conduct.

Our example looks at how individuals define being a good citizen. The *Civic Involvement Survey* conducted by Wuthnow (1997) asked respondents a series of questions set up like this: To be a good citizen, how important is it for people to do each of the following? For each question, respondents provided their answer in one of four categories: essential, very important, fairly important, and not too important. We asked the theoretical question of whether there is simply one dimension of good citizenship, or whether there are multiple dimensions. We consider two possible dimensions: (a) political involvement, captured here by three questions about voting, keeping informed about issues, and contacting public officials; and (b) public service in helping the community, captured here by three questions regard-

[8]This adjustment is necessary as the ACM output by M*plus* must be multiplied by the sample size to put it in an appropriate metric.

ing doing volunteer work, helping the poor, and belonging to a club or service organization. Finally, there is also a question about being a person of strong moral character. It is possible that "moral character" can be interpreted in various ways by respondents, and thus may be affected by both of our dimensions. Those who feel that political involvement is important may view such activity in a moral fashion, and similarly for those who feel voluntary action is important. Therefore, we allow paths from both factors to this measure.

These theorized models are illustrated in the path model of Figure 1. Following conventions, the latent variables are represented as ovals, observed variables are represented as rectangles, and single-headed arrows represent causal paths. The two dashed paths are specifications we add later, but at this point are constrained to values of zero. For the single-factor model, we also constrain the covariance of the two latent variables, $cov(\sigma^2{}_1, \sigma^2{}_2)$, to 1 and the path from the "service" latent variable to y4 to 0. We compare this with the two-factor model that freely estimates these two parameters. Because these dimensions may work differently for members of different ethnic and racial groups, we restrict our attention to African Americans. This gives us a sample size of 227.

We first compare a model treating good citizenship as a single dimension with a model considering the two dimensions of political and public service involvement. To use the CTANEST1 macro, we first obtain the sample covariance or polychoric matrix by specifying any of the models we wish to estimate. Because our data are ordinal, we next obtain the ACM. This ACM and the polychoric matrix are the same for each of the nested tests we are going to perform using CTANEST1; all that changes for each test is the supplied model-implied covariance matrices. For each set of nested models we test, we obtain the model-implied covariance matrices as described earlier in Step 2.

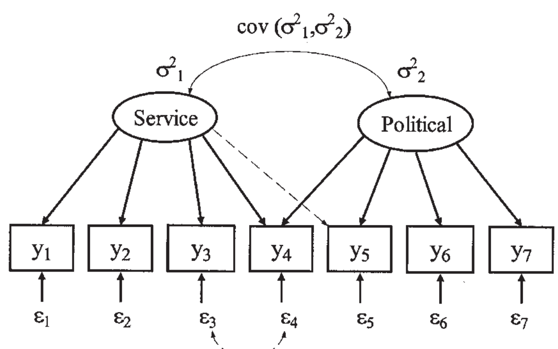

FIGURE 1 Nested model tests for tetrads: One factor, Two factor, Cross-loading, and correlated error models.

Because we have ordinal data, we next read the ACM matrix obtained earlier into a SAS data set (and also the polychoric matrix, if we chose to output that as a file). This was described in Step 4a, showing the code for calling the CTAFILE macro. With these four matrices, we can now enter them into our SAS program that calls the CTANEST1 macro. This was shown in Step 4b. Doing so, we obtain the following output:

```
Tetrad Test for Model with more vanishing tetrads
     Chi-Square degrees of freedom   p-value
     163.19803                  14         0
   Model implied matrices are in correct order
            Models are tetrad nested
Tetrad Test for Model with fewer Vanishing tetrads
     Chi-Square degrees of freedom   p-value
     40.994254                  12 0.0000491
        Nested Tetrad Test for two models
     Chi-Square degrees of freedom   p-value
     122.20378                   2         0
```

The model with more vanishing tetrads is the one-factor model. Note that the fit for this model is quite poor: the χ^2 of 163.2 on 14 *df* is highly significant. The model with fewer vanishing tetrads is the two-factor model. It also does not have a satisfactory fit, with a significant χ^2 of 40.99 on 12 *df.* The nested tetrad test for the two models tests the additional tetrads implied by the single-factor model: Note that the highly significant χ^2 of 122.2 on 2 *df* suggests that the additional vanishing tetrads implied by the single-factor model reduce the fit of the model appreciably. Notice that there are also additional comments telling us that we have entered the model-implied covariance matrices in correct order, and that the models are indeed tetrad-nested.

We next consider the possibility that the question asking about contacting a city official has a cross-loading. That is, although we hypothesized that this would be perceived as a political activity and hence affected by the political dimension of citizenship, it is also possible that some may view this as a public service activity wherein such contact is done to effect change in the conditions of others less fortunate. In Figure 1, the dashed path from the service latent variable to y5 represents this. To test this, we specify a new model with the additional cross-loading, obtain its implied covariance matrix, and enter this along with the implied covariance matrix of our previous two-factor model when calling the CTANEST1 macro. However, for illustration, we entered this model-implied covariance matrix as

IMPMAT1B, telling the program (incorrectly) that this is the model implying more vanishing tetrads. We obtain the following output:

```
          Tetrad Test for Model with more vanishing tetrads
               Chi-Square degrees of freedom   p-value
               39.713042                  12  0.0000802

NOTE: Model implied matrices were incorrectly entered for large and
small models—they have been reversed

                      Models are tetrad nested
         Tetrad Test for Model with fewer Vanishing tetrads
               Chi-Square degrees of freedom   p-value
               21.427673                  10  0.0183011
                  Nested Tetrad Test for two models
               Chi-Square degrees of freedom   p-value
               18.285368                   2  0.000107
```

The program again determines that these two models are tetrad-nested, but also detects that the two implied covariance matrices have been entered in reverse order. It has treated these matrices correctly in the program, and informs us that the first implied covariance matrix actually represents the model with fewer implied vanishing tetrads. Therefore, the model with more vanishing tetrads is the two-factor model without this additional cross-loading. Note that it again has 12 *df,* as in the previous test, but now has a slightly different χ^2. This is because the nested test makes certain that the vanishing tetrads used for the smaller model are a subset of the tetrads in the larger model, and therefore a given nested test may entail a slightly different set of vanishing tetrads from that of another nested test. This illustrates that there is not simply a single set of nonredundant vanishing tetrads for a model, but rather many possible combinations. However, the χ^2 values tend to be fairly similar: 39.7 versus 41.0 in this instance. The model with fewer vanishing tetrads is the one specifying the additional cross-loading, and has a significant χ^2 of 21.4 on 10 *df.* The nested tetrad test shows that the additional two vanishing tetrads implied by the model without this cross-loading significantly reduce the fit of the model, with a χ^2 difference of 18.3 on just 2 *df.* This also illustrates that the number of tetrads implied vanishing by a model need not move monotonically with the degrees of freedom of a likelihood ratio test: In this instance, the single extra parameter would imply a 1 *df* test for the likelihood ratio test, but it implies two fewer tetrads. This nonmonotonic relation points out why the tetrad test can on occasion allow testing models that are not identified for the usual LRT, or are not nested for the LRT.

Finally, there may be reason to suspect that the questions regarding helping the poor and behaving in a moral fashion have an additional relation between them. That is, there may be an additional religious dimension that leads individuals to view helping the poor and behaving morally to contain an additional correlation beyond our specified citizenship dimensions. In Figure 1, the curved double-headed arrow between e_3 and e_4 represents this possible effect. To test this, we include a covariance between the error terms of these two variables. We again specify this new model in an SEM program, obtain the model-implied covariance matrix, and perform our nested tetrad test. We obtain the following output:

```
  Tetrad Test for Model with more vanishing tetrads
       Chi-Square degrees of freedom   p-value
       21.427673                  10  0.0183011
     Model implied matrices are in correct order
              Models are tetrad nested
 Tetrad Test for Model with fewer Vanishing tetrads
       Chi-Square degrees of freedom   p-value
       13.792091                   9  0.1299156
          Nested Tetrad Test for two models
       Chi-Square degrees of freedom   p-value
       7.6355825                   1  0.0057228
```

This new model (with fewer vanishing tetrads), now shows a satisfactory fit, with a nonsignificant χ^2 of 13.8 on 9 *df.* This additional covariance has also improved the model fit over the previous model, as the χ^2 of 7.6 on 1 *df* suggests a significantly worse fit for the model not containing this covariance.

Recall that none of these tests would be appropriate for the researcher using a DWLS approach with a mean and variance-adjusted χ^2 test, as the approximated degrees of freedom precludes such statistical testing. Even with continuous endogenous variables, testing models that are nested by constraining a correlation to the boundary value of one violates the regularity conditions of the LRT. Although this problem is rarely considered when testing such nested structural equation models, the tetrad test provides a viable alternative in such instances.

Finally, now that we have found a satisfactory model, we illustrate the ability of the program to randomize the tetrads. This procedure allows the researcher to assess the robustness of the findings given that a model does not imply a single set of nonredundant vanishing tetrads. To accomplish this, we simply change "reps =" on the macro call line to the value of choice. In this instance, we instructed the program to randomize the tetrads 10 times by changing the value to reps = 10. The program now loops through the test 10 times, and in addition to the usual output, at the bottom we obtain following additional results:

rep	big-chi	big-df	big-p	small-chi	small-df	small-p	test-chi	test-df	test-p
1	20.95	10	0.0215	13.70	9	0.1336	7.25	1	0.0071
2	25.83	10	0.0040	14.96	9	0.0921	10.88	1	0.0010
3	22.72	10	0.0118	14.09	9	0.1193	8.63	1	0.0033
4	24.58	10	0.0062	15.72	9	0.0729	8.86	1	0.0029
5	22.56	10	0.0125	14.62	9	0.1019	7.94	1	0.0048
6	20.23	10	0.0271	13.56	9	0.1388	6.67	1	0.0098
7	21.55	10	0.0176	13.33	9	0.1484	8.22	1	0.0041
8	21.70	10	0.0167	13.79	9	0.1299	7.91	1	0.0049
9	26.43	10	0.0032	15.32	9	0.0825	11.11	1	0.0009
10	20.77	10	0.0228	12.67	9	0.1781	8.10	1	0.0044

The first column shows the randomization number, and the next three columns show the chi-square, degrees of freedom, and *p* value for the model implying more vanishing tetrads. The next three columns show the chi-square, degrees of freedom, and *p* value for the model implying fewer vanishing tetrads, and the last three columns give the same results for the nested test of each randomization. This example shows these results to be rather robust: In virtually every randomization our conclusion regarding each of the individual models and the nested test remain unchanged. This also illustrates how easy it is for applied researchers to assess the robustness of their model results to random draws of the nonredundant vanishing tetrads.

CONCLUSION

The lack of user-friendly software for performing simultaneous tests of vanishing tetrads has lessened the use of these tests by researchers. This article has introduced an SAS macro that allows the researcher to easily employ the vanishing tetrads test in assessing the fit of structural equation models. It allows the applied researcher to take advantage of the many instances in which the vanishing tetrads test has favorable properties, including testing models with dichotomous, ordinal, or censored endogenous variables; some instances in which models are not identified for the usual LRT; some models that are not nested for the LRT; nested tests constraining a parameter to a boundary value; and nested models when using dichotomous, ordinal, or censored endogenous variables.

This macro introduces several features not available in an earlier vanishing tetrad macro (Ting, 1995). First, this program allows the researcher take into account the noncontinuous nature of dichotomous, ordinal, or censored observed variables by using the polychoric matrix when performing the vanishing tetrads test. In such instances, obtaining the polychoric correlation (covariance) matrix and its accompanying asymptotic covariance matrix will allow the researcher to perform such tests. Second, CTANEST1 fully automates the selection of vanishing tetrads in the model. The researcher need not engage in the laborious task of determining which tetrads are close enough to zero in the sample to be considered vanishing in the model, a considerable advantage when testing larger models. Third, given that applied researchers likely do not have an intuitive sense of which models are tetrad-nested, CTANEST1 fully automates the procedure of determining whether various models are indeed nested. Additionally, it then performs the laborious task of selecting the appropriate vanishing tetrads so that the vanishing tetrads of the model implying fewer vanishing tetrads are a subset of the vanishing tetrads in the model implying more vanishing tetrads. Fourth, this macro adopts a new strategy for selecting the vanishing tetrads implied by the model using the sweep operator. Hipp and Bollen (2003) pointed out that this strategy is about 60 times faster than the procedure suggested by Bollen and Ting (1993). Finally, the program has fully automated the procedure of pulling random draws of vanishing tetrads when assessing the fit of models. Because there is generally not a unique set of vanishing tetrads for any given model, Hipp and Bollen (2003) suggested that the researcher may wish to pull random draws of vanishing tetrads to assess the sensitivity of the results to such selection. Our experience has been that the choice of vanishing tetrads makes little difference. However, this does allow the researcher to assess the robustness of the tetrad test results. The result is a macro that allows the applied researcher to easily utilize the many advantages of the vanishing tetrads test.

Finally, the ability of the CTANEST1 macro to randomize the tetrads used will facilitate future simulation studies of the behavior of the vanishing tetrads test in finite samples. This is advantageous, for although the limited simulation work on the vanishing tetrads test has shown it to perform fairly satisfactorily, more work clearly needs to be done. Bollen and Ting (1998) showed that the combination of model size and sample size was important for the performance of the test. For smaller models, the test performed favorably on continuous data with samples of 300 and larger. For smaller sample sizes a bootstrapping technique they proposed resulted in a more satisfactory rejection rate. More recently, Hipp and Bollen (2003) showed in a small simulation that the vanishing tetrads test also performs quite well when using the polychoric matrix for dichotomous, ordinal, or censored variables. The test exhibited very satisfactory performance for various sample sizes, various distributions of the ordinal data, and violations of the assumption of underlying normality of the observed variables. Additionally, it nearly always performed more favorably than WLS and generally performed as well or better than DWLS with a mean and variance-adjusted chi-square. Given this latter result, and the vanishing tetrads test's ability to test nested models (a feature not available for DWLS with a mean and variance-adjusted chi-square), it seems to be a particularly viable option when assessing the fit of models for noncontinuous data. Although such results suggest the viability of the vanishing tetrads test, the usual caveat of the limited generalizability of simulation studies holds in this instance and suggests a need for further probing of the performance of this test.

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