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Testing Main Effects and Interactions in Latent Curve Analysis

Patrick J. Curran, Daniel J. Bauer, and Michael T. Willoughby
University of North Carolina at Chapel Hill

A key strength of latent curve analysis (LCA) is the ability to model individual variability in rates of change as a function of 1 or more explanatory variables. The measurement of time plays a critical role because the explanatory variables multiplicatively interact with time in the prediction of the repeated measures. However, this interaction is not typically capitalized on in LCA because the measure of time is rather subtly incorporated via the factor loading matrix. The authors' goal is to demonstrate both analytically and empirically that classic techniques for probing interactions in multiple regression can be generalized to LCA. A worked example is presented, and the use of these techniques is recommended whenever estimating conditional LCAs in practice.

Random-effects growth models have become increasingly popular in applied behavioral and social science research. The two primary approaches used for estimating these models are the hierarchical linear model (HLM; Bryk & Raudenbush, 1987; Raudenbush & Bryk, 2002) and structural equation-based latent curve analysis (LCA; Meredith & Tisak, 1984, 1990).[1] The variable measuring the passage of time plays a critical role in both the HLM and LCA approaches, although the way in which this measure is incorporated into the model is quite different. The HLM approach explicitly incorporates the measure of time as an exogenous predictor variable within the Level 1, or person-level, equation. In contrast, the LCA approach incorporates the measure of time by placing specific restrictions on the values of the factor loading matrix that relate the repeated measures to the underlying latent growth factors. In many situations these two approaches to growth modeling are analytically equivalent, whereas in other situations they are not (e.g., MacCallum, Kim, Malarkey, & Kiecolt-Glaser, 1997; Willett & Sayer, 1994).

Even under the conditions in which the HLM and LCA approaches provide equivalent results, there are subtle but important differences in model estimation and interpretation that arise from the different incorporation of time. These differences are primarily manifested when conditional growth models—that is, models that include one or more exogenous variables that predict the random growth curve parameters—are considered. In both the HLM and LCA approaches, main effect predictions of the random trajectories imply that the exogenous variables interact with time in the prediction of the repeated measures. In HLM, both the predictors and time are treated as exogenous variables in the model, and the interaction between them is explicitly represented as a cross-level interaction (see, e.g., Bryk & Raudenbush, 1987; Curran, Bauer, & Willoughby, in press; Willett, Singer, & Martin, 1998). This representation has facilitated the occasional use of plotting interactions to aid in the interpretation of complex HLM growth model results (e.g., Bryk & Raudenbush, 1987, p. 154; Singer, 1998, p. 345; Willett et al., 1998, p. 423). In contrast, in LCA time is not treated as a variable at all but is instead incorporated into the model via the factor loading matrix. As such, the interaction between time and other predictors is not readily apparent. Instead, the multiplicative interaction between exogenous predictors and time is captured in LCA by the indirect effects of the predictors on the repeated measures as

Patrick J. Curran, Daniel J. Bauer, and Michael T. Willoughby, Department of Psychology, University of North Carolina at Chapel Hill.

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Sample data, computer code, and a Web-based interface for computing simple slopes and regions of significance can be downloaded from http://www.unc.edu/~curran.

Correspondence should be addressed to Patrick J. Curran, Department of Psychology, University of North Carolina at Chapel Hill, Chapel Hill, NC 27599-3270. E-mail: curran@unc.edu

[1] We use the term *latent curve analysis* to remain consistent with the original work of Meredith and Tisak (1990). Furthermore, we use the terms *curve*, *growth curve*, and *trajectory* interchangeably throughout the text.

conveyed through the factor loading matrix.[2] Thus the parameterization of the growth model in LCA, though analytically equivalent to the HLM in many cases, tends to obscure the role of time as a predictor in the model. In fact, we are aware of no published applications that have explicitly investigated the interaction between exogenous predictors and time within the LCA framework (but see Choi & Hancock, 2003, for an initial discussion of this possibility).

We argue in this article that time can and should be treated as a predictor in the LCA growth model just as it is in the HLM framework even though time is incorporated through the factor loading matrix. When viewed in this way, main effect predictors of the growth factors are seen to interact with time in the prediction of the repeated measures. Further, two-way interactions between predictors of the latent curve factors are, in turn, three-way interactions with time, as are all higher order interactions. We demonstrate that these interactions can be explicitly evaluated within LCA by reconceptualizing the indirect effects of the predictors on the repeated measures through the factor loading matrix as multiplicative interactions with time. By doing this, we can bring to bear techniques for testing and probing interactions that allow us to exploit the richness of these models in a way that would not otherwise be possible. Our motivating goal for this article is to analytically extend the use of traditional techniques for exploring interactive effects to the LCA framework and to demonstrate the benefits of their use with an empirical example.

We begin our article with a brief review of the estimation and interpretation of the standard unconditional and conditional latent curve model. Next, we discuss the importance of closely considering conditional effects whenever testing exogenous predictors of the latent growth factors. We then describe methods for estimating and testing simple slopes associated with both categorical and continuous exogenous predictors, and we extend these tests to calculate regions of significance and confidence bands. We then generalize all of these methods to include higher order interactions among two or more exogenous predictors of the latent growth factors. We briefly consider another method for exploring interactions that makes use of the unique features of the structural equation model (SEM)—namely, the multiple-groups model. We present an empirical demonstration of our proposed methods using data from a cohort-sequential longitudinal sample of children with unbalanced data, and we conclude with potential limitations and directions for future research.

The Unconditional Latent Curve Model

The latent curve model is, at its core, a factor analysis model. Early work in this area involved principal-components analysis of the sums of squares and cross products of the repeated measures (e.g., Rao, 1958; Tucker, 1958). Meredith and Tisak (1984, 1990) later demonstrated the advantages of a confirmatory factor analysis (CFA) model for estimating patterns of individual growth. This model was further expanded in important ways by McArdle (1988, 1989, 1991), Muthén (1991, 1993), and others.

We can define y_{it} to represent a continuous repeated measure of construct y assessed on a sample of $i = 1, 2, \ldots, N$ individuals at $t = 1, 2, \ldots, T_i$ time points. In matrix notation, the measurement model of the CFA takes the form

$$\mathbf{y} = \boldsymbol{\nu} + \boldsymbol{\Lambda}\boldsymbol{\eta} + \boldsymbol{\varepsilon}, \qquad (1)$$

where $\mathbf{y}$ is a $T_i \times 1$ vector of repeated measures for individual i, $\boldsymbol{\nu}$ is $T_i \times 1$ vector of measurement intercepts, $\boldsymbol{\Lambda}$ is a $T_i \times k$ matrix of factor loadings, $\boldsymbol{\eta}$ is a $k \times 1$ vector of latent curve factors, and $\boldsymbol{\varepsilon}$ is a $T_i \times 1$ vector of time-specific residuals. Because $\boldsymbol{\nu} \equiv \mathbf{0}$ for identification purposes in the class of models discussed here, we do not consider this vector further.

In most factor analysis models the elements in $\boldsymbol{\Lambda}$ are freely estimated from the data. However, in latent curve models these elements are often fixed to predetermined values to specify a particular functional form for the growth process (although other nonlinear forms are possible; see, e.g., Browne, 1993; du Toit & Cudeck, 2001). For example, the elements of Equation 1 for a linear latent curve model for T_i measurement occasions are

$$\begin{bmatrix} y_{i1} \\ y_{i2} \\ \vdots \\ y_{iT_i} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ \vdots & \vdots \\ 1 & T_i - 1 \end{bmatrix} \begin{bmatrix} \eta_{\alpha_i} \\ \eta_{\beta_i} \end{bmatrix} + \begin{bmatrix} \varepsilon_{i1} \\ \varepsilon_{i2} \\ \vdots \\ \varepsilon_{iT_i} \end{bmatrix}. \qquad (2)$$

The measure of time is thus entered into the model via the factor loading matrix $\boldsymbol{\Lambda}$. This is highlighted in the scalar expression of Equation 2 for individual i,

$$y_{it} = \eta_{\alpha_i} + \eta_{\beta_i}\lambda_{it} + \varepsilon_{it}, \qquad (3)$$

indicating that the observed score on measure y for individual i at time t is an additive function of the intercept of the trajectory for individual i (e.g., η_{α_i}), the slope of the trajectory for individual i multiplied by the value of time for individual i at time t (e.g., $\eta_{\beta_i}\lambda_{it}$), and an individual- and time-specific residual (e.g., ε_{it}). This corresponds to the

[2] One might wonder about the possibility of using methods for decomposing direct and indirect effects in more standard structural equation models (e.g., Bollen, 1987) for probing interactions in LCA. Although these same methods can be applied here, no new information is available in the most common LCA application in which the factor loadings are fixed to predefined values (and thus do not have associated standard errors). Some interesting tests of indirect effects can be conducted when one or more of the factor loadings are freely estimated from the data, but given space constraints, we do not pursue these here.

typical "Level 1" equation in hierarchical linear modeling. If all individuals are assessed at the same time periods (i.e., a "balanced" longitudinal design is used), then the value of time at assessment t is λ_t for all i; if the design is not balanced, then λ_{it} represents the value of time at t that is specific to individual i. All of the methods we describe here apply to the use of either λ_t or λ_{it}. Thus, for simplicity, we drop the i subscript and simply use λ_t hereafter.

Because the latent factors in $\boldsymbol{\eta}$ are random, these can be expressed as

$$\boldsymbol{\eta} = \boldsymbol{\mu}_\eta + \boldsymbol{\zeta}, \qquad (4)$$

where $\boldsymbol{\mu}_\eta$ is a $k \times 1$ vector of latent variable means and $\boldsymbol{\zeta}$ is a $k \times 1$ vector of individual deviations from these means. For the linear latent curve model, the elements of Equation 4 are

$$\begin{bmatrix} \eta_{\alpha_i} \\ \eta_{\beta_i} \end{bmatrix} = \begin{bmatrix} \mu_\alpha \\ \mu_\beta \end{bmatrix} + \begin{bmatrix} \zeta_{\alpha_i} \\ \zeta_{\beta_i} \end{bmatrix} \qquad (5)$$

with corresponding scalar expressions

$$\eta_{\alpha_i} = \mu_\alpha + \zeta_{\alpha_i}, \qquad (6)$$
$$\eta_{\beta_i} = \mu_\beta + \zeta_{\beta_i}.$$

Thus, Equation 6 indicates that the individual-specific intercept and individual-specific slope are an additive function of the mean intercept and slope and an individual-specific deviation from these mean values. This corresponds to the typical "Level 2" expression in hierarchical linear modeling.

We can substitute Equation 4 into Equation 1 to express the reduced-form equation for $\mathbf{y}$ such that

$$\mathbf{y} = (\boldsymbol{\Lambda}\boldsymbol{\mu}_\eta) + (\boldsymbol{\Lambda}\boldsymbol{\zeta} + \boldsymbol{\varepsilon}), \qquad (7)$$

where the first and second parenthetical terms are often referred to as the fixed- and random-effect components of the model. The model-implied covariance structure of $\mathbf{y}$ as a function of model parameters in vector $\boldsymbol{\theta}$ is

$$\boldsymbol{\Sigma}(\boldsymbol{\theta}) = \boldsymbol{\Lambda}\boldsymbol{\Psi}\boldsymbol{\Lambda}' + \boldsymbol{\Theta}_\varepsilon, \qquad (8)$$

where $\boldsymbol{\Theta}_\varepsilon$ represents the $T \times T$ covariance matrix of the residuals for the T repeated measures of $\mathbf{y}$ and $\boldsymbol{\Psi}$ represents the $k \times k$ covariance matrix of the k latent curve factors. Finally, the model-implied mean structure of the latent curve model is

$$\boldsymbol{\mu}(\boldsymbol{\theta}) = \boldsymbol{\Lambda}\boldsymbol{\mu}_\eta, \qquad (9)$$

where $\boldsymbol{\mu}(\boldsymbol{\theta})$ is a $T \times 1$ vector of means of $\mathbf{y}$ and where $\boldsymbol{\mu}_\eta$ and $\boldsymbol{\Lambda}$ are defined as before. Of importance, again considering the linear trajectory model, Equation 9 allows us to express the model-implied mean of y at time t as

$$\mu_{y_t} = \mu_\alpha + \mu_\beta\lambda_t. \qquad (10)$$

This equation highlights that the relation between the repeated measures and the latent curve factors operates through the factor loading matrix. We capitalize on this relation extensively later in the article.

The Conditional Latent Curve Model

The unconditional latent curve model allows for important inferences to be made about the mean trajectory for the group (i.e., the fixed effects) and variability in individual trajectories around these mean values (i.e., the random effects). A common next step is to incorporate one or more exogenous predictors of these random trajectories. We can expand Equation 4 to include one or more exogenous predictors of the latent curve factors such that

$$\boldsymbol{\eta} = \boldsymbol{\mu}_\eta + \boldsymbol{\Gamma}\mathbf{x} + \boldsymbol{\zeta}, \qquad (11)$$

where $\mathbf{x}$ is a $p \times 1$ vector of exogenous predictors and $\boldsymbol{\Gamma}$ is a $k \times p$ matrix of fixed regression parameters between the k latent curve factors and the p exogenous predictors in $\mathbf{x}$. A path diagram with two exogenous predictors of a linear latent curve model for $T = 4$ is presented in Figure 1. Given the regression of $\boldsymbol{\eta}$ on $\mathbf{x}$ via $\boldsymbol{\Gamma}$, $\boldsymbol{\mu}_\eta$ now contains the intercepts of the latent curve factors (e.g., the mean of the

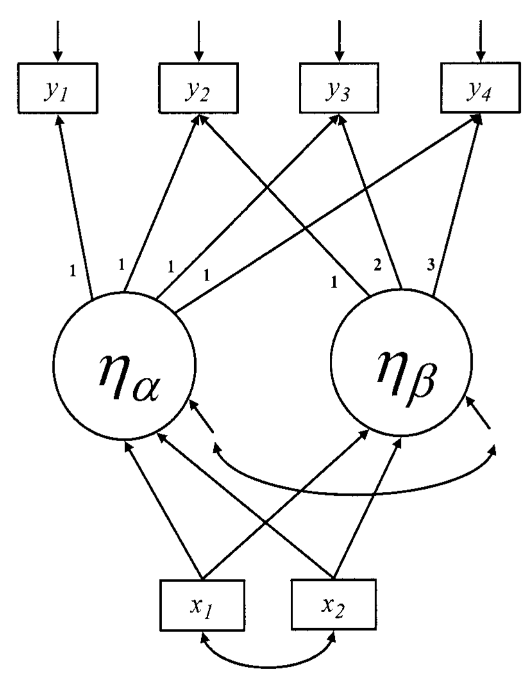

Figure 1. Path diagram of a conditional linear latent curve model for four repeated measures and two correlated exogenous predictor variables; η_α and η_β represent the latent intercept and latent slope of the trajectory, respectively.

latent curve factors at $\mathbf{x} = 0$) and $\boldsymbol{\zeta}$ contains the individually varying residuals.

We can again create a reduced-form expression by substituting Equation 11 into Equation 1 such that

$$\mathbf{y} = (\boldsymbol{\Lambda}\boldsymbol{\mu}_\eta + \boldsymbol{\Lambda}\boldsymbol{\Gamma}\mathbf{x}) + (\boldsymbol{\Lambda}\boldsymbol{\zeta} + \boldsymbol{\varepsilon}). \tag{12}$$

The model-implied covariance matrix for $\mathbf{y}$ is then given as

$$\boldsymbol{\Sigma}(\boldsymbol{\theta}) = \boldsymbol{\Lambda}(\boldsymbol{\Gamma}\boldsymbol{\Phi}\boldsymbol{\Gamma}' + \boldsymbol{\Psi})\boldsymbol{\Lambda}' + \boldsymbol{\Theta}_\varepsilon, \tag{13}$$

where $\boldsymbol{\Phi}$ represents the $p \times p$ covariance matrix of the p exogenous predictors in $\mathbf{x}$ and the other matrices are defined as earlier. The expected value of this reduced-form expression is

$$\boldsymbol{\mu}(\boldsymbol{\theta}) = (\boldsymbol{\Lambda}\boldsymbol{\mu}_\eta + \boldsymbol{\Lambda}\boldsymbol{\Gamma}\boldsymbol{\mu}_x), \tag{14}$$

where $\boldsymbol{\mu}_x$ represents the $p \times 1$ vector of means of exogenous measures. Without loss of generality, we can set $\boldsymbol{\mu}_x = 0$ (i.e., all exogenous measures are mean deviated), and Equation 14 simplifies to Equation 9. We make use of this simplification later.

The Importance of Assessing Conditional Effects in LCA

Why is it important to consider main effect predictors of the slope factors as interactions with time? Consider the following example. Say that we estimated an unconditional latent curve model that reflected a significant negative trajectory over time. Further say that we regressed this slope factor on a continuous predictor and found a significant positive regression coefficient. This would imply that higher values of the predictor variable are associated with larger values of the slope of the trajectory. However, as the value of the predictor increases, the corresponding increase in the value of the slope factor may imply conditional slopes that are negative and significantly different from zero, near zero but not significantly different from zero, or positive and significantly different from zero. The significant positive regression coefficient between the predictor variable and the slope factor is necessary but not sufficient information to explicate these more complex effects. Only through the further probing of this effect can we fully understand the nature of this relation.

To highlight this, consider the simple case in which the intercept and linear slope factors are regressed on a single time-invariant exogenous variable denoted x_i (which may be categorical or continuous). The scalar expressions for these regressions (presented in matrix form in Equation 11) are

$$\eta_{\alpha_i} = \mu_\alpha + \gamma_1 x_i + \zeta_{\alpha_i},$$
$$\eta_{\beta_i} = \mu_\beta + \gamma_2 x_i + \zeta_{\beta_i}, \tag{15}$$

and the scalar reduced-form expression (presented in matrix form in Equation 12) is

$$y_{it} = (\mu_\alpha + \mu_\beta\lambda_t + \gamma_1 x_i + \gamma_2\lambda_t x_i) + (\zeta_{\alpha_i} + \zeta_{\beta_i}\lambda_t + \varepsilon_{it}). \tag{16}$$

Of importance, Equation 16 highlights that a 1-unit change in the exogenous predictor x is associated with a γ_1-unit change in the expected value for the trajectory intercept and a γ_2-unit change in the expected value of the slope of the trajectory. However, the increment to the slope factor in turn influences the repeated observations of y via λ_t. That is, the magnitude of the influence of x in the prediction of y depends on the particular point in time λ_t. In the SEM tradition, $\gamma_2\lambda_t$ would normally be conceptualized as a *compound coefficient* reflecting the indirect effect of the predictor on the repeated measures. However, this conceptualization may obscure the special role of time as a predictor in LCA because time is not represented explicitly as a variable influencing the repeated measures and instead appears only in the factor loading matrix. Given that λ_t is a proxy for the time variable, we may view γ_2 as the coefficient of the multiplicative interaction of the predictor with time.

Reconceptualizing the effects of exogenous predictors as multiplicative interactions not only explicates the role of time in the latent curve model but also encourages the use of available methods for further probing this interaction in important ways. By *probe*, we mean the calculation of point estimates along with standard errors and confidence intervals that provide inferential tests of the relation between the repeated measures and time conditioned on one or more exogenous predictor variables. There are three procedures that are of particular importance here: tests of *simple slopes* (Cohen & Cohen, 1983), Johnson–Neyman tests of *regions of significance* (Johnson & Neyman, 1936; Pothoff, 1964; Rogosa, 1980, 1981), and computation of *confidence bands* for the conditional effects (Rogosa, 1980, 1981). Although these tests are well developed for standard regression models (see Aiken & West, 1991), we are not aware of any prior work that has extended these tests to the latent curve modeling framework. This is our goal here.

Estimating and Testing Simple Slopes

As a starting point, let us again consider the case in which we have a single predictor x of the intercept and slope of a linear latent curve model. The reduced-form equation for the model is then given in Equation 16. Taking expectations of this expression shows that the conditional mean of y varies as a function of both x and λ_t such that

$$\mu_{y_t|x} = \mu_\alpha + \mu_\beta\lambda_t + \gamma_1 x + \gamma_2\lambda_t x. \tag{17}$$

If we treat λ_t as a proxy for the variable time, then we can clearly see that this equation is parallel in form to a regression model including an intercept, main effect of time, main effect of x, and interaction between time and x. To highlight the role of λ_t as a moderator of the regression of y on x, we can rewrite this equation as

$$\mu_{y_t}|_{\lambda_t} = (\mu_\alpha + \mu_\beta\lambda_t) + (\gamma_1 + \gamma_2\lambda_t)x, \tag{18}$$

so that the effect of x is clearly expressed as a function of time ($\gamma_1 + \gamma_2\lambda_t$). Of course, given the symmetry of the interaction, we might also be interested in the role of x in moderating the value of the expected trajectory slope (or fixed effect of time), which is highlighted by rewriting this equation as

$$\mu_{y_t}|_x = (\mu_\alpha + \gamma_1 x) + (\mu_\beta + \gamma_2 x)\lambda_t. \tag{19}$$

In standard regression models, one tool used to better understand conditional relationships of this sort is to plot the effect of one predictor at specific levels of the moderating variable (Cohen & Cohen, 1983). Referring to these conditional effects as *simple slopes,* Aiken and West (1991) showed the utility of not only plotting simple slopes but also testing their statistical significance. These same tests of simple slopes can be extended directly to the latent curve model.

The only difference between the simple regression case and the latent curve model is that there are additional variance components in the LCA—that is, individual variability about μ_α and μ_β. However, the additional variance components associated with these random effects impact minimally on the computational formulas for the simple slopes and their standard errors because we are operating at the level of the fixed effects (i.e., our concern is with $\mu_{y_t}|_x$, not y_{it}). In fact, the only impact of these additional variance components is in the estimation of the standard errors of the parameter estimates, which in turn influence the standard errors and tests of significance of the simple slopes.

For the one-predictor case, we can express the simple slope of the regression of y on x (e.g., the effect of x at a specific value of λ_t) as

$$\hat{\gamma}_1|_{\lambda_t} = \hat{\gamma}_1 + \hat{\gamma}_2\lambda_t. \tag{20}$$

Standard errors for $\hat{\gamma}_1|_{\lambda_t}$ can be derived using standard covariance algebra, resulting in

$$SE(\hat{\gamma}_1|_{\lambda_t}) = [VAR(\hat{\gamma}_1) + 2\lambda_t COV(\hat{\gamma}_1, \hat{\gamma}_2) + \lambda_t^2 VAR(\hat{\gamma}_2)]^{1/2}, \tag{21}$$

where *VAR* and *COV* represent the appropriate variance and covariance elements from the asymptotic covariance matrix of parameter estimates (see the Appendix for further details).[3] If we use a z distribution for large samples, the test statistic is then

$$z_{\hat{\gamma}_1|\lambda_t} = \frac{\hat{\gamma}_1|_{\lambda_t}}{SE(\hat{\gamma}_1|_{\lambda_t})}. \tag{22}$$

However, given the symmetry of the interaction, we may be equally (if not more) interested in the estimated values of the simple intercept and slope of the latent trajectory at specific values of x:

$$\hat{\mu}_\alpha|_x = \hat{\mu}_\alpha + \hat{\gamma}_1 x,$$
$$\hat{\mu}_\beta|_x = \hat{\mu}_\beta + \hat{\gamma}_2 x. \tag{23}$$

Consistent with our interest in model-implied trajectories, we refer to these terms collectively as the *simple trajectory* at the chosen level of x. Standard errors for these sample estimates can be derived using covariance algebra, resulting in

$$SE(\hat{\mu}_\alpha|_x) = [VAR(\hat{\mu}_\alpha) + 2xCOV(\hat{\mu}_\alpha, \hat{\gamma}_1) + x^2 VAR(\hat{\gamma}_1)]^{1/2},$$
$$SE(\hat{\mu}_\beta|_x) = [VAR(\hat{\mu}_\beta) + 2xCOV(\hat{\mu}_\beta, \hat{\gamma}_2) + x^2 VAR(\hat{\gamma}_2)]^{1/2}, \tag{24}$$

where *VAR* and *COV* again represent the appropriate variance and covariance elements from the asymptotic covariance matrix of parameter estimates. Note that these elements explicitly take into account the presence of the random effects in the model via the estimation process. Further, with the standard errors in hand, significance tests of the intercepts and slopes of the simple trajectories can be conducted in the usual way. If we use a z distribution for large samples, the test statistics are

$$z_{\hat{\mu}_\alpha} = \frac{\hat{\mu}_\alpha|_x}{SE(\hat{\mu}_\alpha|_x)},$$
$$z_{\hat{\mu}_\beta} = \frac{\hat{\mu}_\beta|_x}{SE(\hat{\mu}_\beta|_x)}. \tag{25}$$

These point estimates, standard errors, and z tests apply to any main effect predictor x. We next briefly explicate the specific applications of these methods for categorical and continuous covariates.

Categorical Covariates

Suppose that our predictor is a dummy-coded categorical variable in which $x = 0$ and $x = 1$ denote membership in one of two discrete groups (e.g., gender, treatment condi-

[3] Note that we are referring to the sample estimate of the asymptotic covariance matrix of parameter estimates for all of our developments and applications.

tion). We could write the simple trajectories for each of the two groups as

$$\mu_{y_t}|_{x=0} = \mu_\alpha + \mu_\beta \lambda_t,$$

$$\mu_{y_t}|_{x=1} = (\mu_\alpha + \gamma_1) + (\mu_\beta + \gamma_2)\lambda_t, \tag{26}$$

where the first parenthetical term in each expression is the intercept of the simple trajectory and the second term is the slope of the simple trajectory within that specific group.

Equation 26 highlights several important aspects of the conditional latent curve model. First, μ_α and μ_β represent the intercept and slope of the simple trajectory when the predictor equals zero (i.e., the mean intercept and slope for group $x = 0$). Further, γ_1 reflects the difference between the mean intercept for group $x = 1$ compared with group $x = 0$, and γ_2 reflects the difference between the mean slope for group $x = 1$ compared with group $x = 0$. Thus, although we have a formal test of the difference in mean slopes between the two groups, we do not yet have a point estimate or standard error for the conditional trajectory within the second group. Our goal is to compute the point estimates and corresponding standard errors for the simple trajectory within group $x = 0$ and the simple trajectory within group $x = 1$.

One way of doing so is to apply the Equation 24 directly. However, we can equivalently compute these point estimates and standard errors using any standard SEM software package. To see this, note that for $x = 0$, Equation 24 simplifies to

$$SE(\hat{\mu}_\alpha|_{x=0}) = VAR(\hat{\mu}_\alpha)^{1/2},$$

$$SE(\hat{\mu}_\beta|_{x=0}) = VAR(\hat{\mu}_\beta)^{1/2}, \tag{27}$$

indicating that the point estimates and standard errors for the simple trajectory in the group denoted $x = 0$ are the intercept terms for the intercept and slope equations. We can capitalize on this to compute the point estimates and standard errors for the group $x = 1$ without recourse to the computational formulas above. To do so, we simply reverse the dummy coding of group membership and reestimate the model. Of course, the overall fit of the second model will be identical to that of the first, but the resulting point estimates for $\hat{\mu}_\alpha$ and $\hat{\mu}_\beta$ and their standard errors will represent the estimated simple trajectory of the second group and will be equivalent to those calculated by the formulas given above. (See Aiken & West, 1991, for a more detailed discussion of this approach as applied to the standard regression model.)

Continuous Covariates

The only difference between categorical and continuous predictors is that in the continuous case there are typically no natural levels at which to assess simple trajectories. Cohen and Cohen (1983) suggested choosing high, medium, and low values of the predictor, often defined as the mean and the mean ± 1 standard deviation. Using these values, we can then apply Equation 24 to obtain the standard errors for any given value of *x*. However, just as in the dichotomous case, these same point estimates and standard errors can be obtained using any standard SEM software package by rescaling the predictor and reestimating the model. This is facilitated by using centered predictors, because the initial model estimation will then produce the estimates and standard errors for the simple trajectory at the mean. First, we create two new variables that are linear transformations of the original predictor, such that $x_{\text{high}} = x - SD_x$ and $x_{\text{low}} = x + SD_x$, where SD_x is the standard deviation of *x*. (As a brief aside, note that it is correct that 1 *SD* is subtracted to compute x_{high} and that 1 *SD* is added to compute x_{low}. This is because we take advantage of the fact that the intercepts of the regression equations predicting the intercept and slope factors represent the model-implied mean when all predictors are equal to zero. When we add 1 *SD* to all *x* scores, a value of zero on *x* represents 1 *SD* below the mean, and vice versa.) Finally, we estimate two separate models in which we use x_{high} and x_{low} in place of our original *x;* the parameter estimates and standard errors for the intercept terms of the intercept and slope equations are equal to the values that would be obtained using Equations 23 and 24.

Regions of Significance

Using the above procedures, we can evaluate the simple slopes of the effect of *x* predicting *y* for specific points in time (e.g., Equation 20), or we can evaluate the simple trajectories of the effect of time predicting *y* for specific levels of *x* (e.g., Equation 23). We now show how we can gain even further information about the conditional effects of *x* and time by computing regions of significance, a technique originally applied in regression models by Johnson and Neyman (1936) with subsequent extensions by Pothoff (1964) and Rogosa (1980).[4] Regions of significance will allow us to assess at precisely what periods of time the simple slopes of *x* predicting *y* pass from significance to nonsignificance. Similarly, regions of significance will allow us to assess at precisely what points on the scale of *x* (when *x* is continuous) the simple trajectories of time predicting *y* pass from significance to nonsignificance. We address each of these in turn.

We begin by asking the question, Over what range of time is the effect of *x* predicting *y* significant? Our interest is in

[4] Pothoff (1964) distinguished between *simultaneous* and *nonsimultaneous* regions of significance. For ease of presentation we only focus on nonsimultaneous regions here, although the computation of simultaneous regions is easily obtained (see Pothoff, 1964, Equation 3.1). The same distinction applies in the subsequent discussion of confidence bands, and again we focus on nonsimultaneous confidence bands.

finding the specific values of time, where time is treated as a continuous variable, for which the simple slope of the effect of x is significantly negative, nonsignificant, or significantly positive. To determine this, we reverse the procedure for computing the significance of the simple slopes. Specifically, when testing simple slopes, we choose a specific value of λ_t, compute the point estimate and standard error for the simple slope at that value of λ_t, and then solve Equation 22 for $z_{\hat{\gamma}_1|_\lambda t}$. When computing regions of significance, we instead choose a specific critical value for the test statistic, say ± 1.96 for $\alpha = .05$, and then solve Equation 22 for the specific values of λ_t that yield this critical value. These values of λ_t indicate the points in time at which $z_{\hat{\gamma}_1|_\lambda t}$ passes from significance to nonsignificance, thus demarcating the regions of significance.

This is accomplished computationally by substituting the critical value of ± 1.96 for $z_{\hat{\gamma}_1|_\lambda t}$ in Equation 22 and also substituting Equations 20 and 21 for $\hat{\gamma}_1|_{\lambda_t}$ and $SE(\hat{\gamma}_1|_{\lambda_t})$, respectively. Squaring this expression and performing a few algebraic rearrangements (see the Appendix), we have

$$[z^2_{\text{crit}}VAR(\hat{\gamma}_2) - \hat{\gamma}_2^2]\lambda_t^2 + \{2[z^2_{\text{crit}}COV(\hat{\gamma}_1, \hat{\gamma}_2) - \hat{\gamma}_1\hat{\gamma}_2]\}\lambda_t + [z^2_{\text{crit}}VAR(\hat{\gamma}_1) - \hat{\gamma}_1^2] = 0, \quad (28)$$

where z_{crit} is the critical value for the z statistic (e.g., ± 1.96). We can then solve for the roots of λ_t that satisfy this equality using the quadratic formula,

$$\lambda_t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \quad (29)$$

where

$$a = [z^2_{\text{crit}}VAR(\hat{\gamma}_2) - \hat{\gamma}_2^2], \quad (30)$$

$$b = \{2[z^2_{\text{crit}}COV(\hat{\gamma}_1, \hat{\gamma}_2) - \hat{\gamma}_1\hat{\gamma}_2]\}, \quad (31)$$

$$c = [z^2_{\text{crit}}VAR(\hat{\gamma}_1) - \hat{\gamma}_1^2]. \quad (32)$$

The resulting roots indicate the boundaries of the regions of significance of x predicting y over all possible values of time.

We next ask the question, Over what range of the predictor x is the effect of time predicting y significant? To answer this, we apply regions of significance to the intercept and slope estimates for the simple trajectories across values of x. This procedure will generally only be useful when x is a continuous covariate, as it is in this case that we might be interested in a range of values for x over which the simple trajectories are significantly different from zero. The procedures we follow to define the regions of significance for the simple trajectories are precisely the same as those illustrated above for the simple slopes of x predicting y as a function of time. Namely, we select a value z_{crit} and solve for the values of x that yield z_{crit} in the formulas of Equation 25. This will again result in a quadratic equation that can be solved using the quadratic formula (see the Appendix for detail). The resulting roots will indicate the specific regions of x over which the intercepts and slopes of the simple trajectories are or are not significantly different from zero. This is a substantial improvement over the arbitrary selection of high, medium, and low values at which to assess the simple trajectories, as the regions indicate the significance of the simple trajectories over all possible values of x.

Confidence Bands

Both tests of simple slopes and regions of significance are based on traditional null hypothesis-testing procedures. However, the usefulness of this approach to statistical inference has been questioned repeatedly over the decades, culminating in the APA task force report emphasizing that the construction of confidence intervals is often better than simple null hypothesis tests (Wilkinson & American Psychological Association Task Force on Statistical Inference, 1999). One reason for this recommendation is that confidence intervals provide more information than null hypothesis tests. For instance, the same test of the null hypothesis is accomplished by determining whether the confidence interval encloses zero. More important, there is no need to establish a specific value (usually zero) for the null hypothesis, as one can select any values of interest and see immediately whether they lie within or without the confidence interval. Further, the breadth of the confidence interval conveys the degree of certainty in our point estimate of the population parameter, often indicating considerable imprecision even when very small probability values are obtained from simple null hypothesis tests. It is for these reasons that we present a third technique for evaluating conditional effects: confidence bands.

We begin with the standard formula for a confidence interval for a given parameter estimate $\hat{\theta}$ where

$$CI = \hat{\theta} \pm z_{\text{crit}}[SE(\hat{\theta})]. \quad (33)$$

In most cases we are interested in a single effect estimate and so simply compute the confidence interval for this estimate. However, in the case of conditional effects, both the effect estimate and its standard error vary as a function of the moderating variable. As such, we cannot plot just one confidence interval; instead we must plot the confidence interval at each level of the moderating variable, what are known as *confidence bands.*

To compute confidence bands for the effect of x conditional on λ_t, we substitute Equations 20 and 21 for the corresponding terms in Equation 33:

$$CB_{\hat{\gamma}_1|_{\lambda_t}} = (\hat{\gamma}_1 + \hat{\gamma}_2\lambda_t) \pm z_{\text{crit}}\{[VAR(\hat{\gamma}_1) + 2\lambda_t COV(\hat{\gamma}_1, \hat{\gamma}_2) + \lambda_t^2 VAR(\hat{\gamma}_2)]^{1/2}\}. \quad (34)$$

Similar procedures yield the confidence bands for the intercept and slope estimates of the simple trajectories

$$CB_{\hat{\mu}_\alpha|x} = (\hat{\mu}_\alpha + \hat{\gamma}_1 x) \pm z_{\text{crit}}[VAR(\hat{\mu}_\alpha) + 2xCOV(\hat{\mu}_\alpha, \hat{\gamma}_1) + x^2 VAR(\hat{\gamma}_1)]^{1/2},$$

$$CB_{\hat{\mu}_\beta|x} = (\hat{\mu}_\beta + \hat{\gamma}_2 x) \pm z_{\text{crit}}[VAR(\hat{\mu}_\beta) + 2xCOV(\hat{\mu}_\beta, \hat{\gamma}_2) + x^2 VAR(\hat{\gamma}_2)]^{1/2}. \quad (35)$$

As is the case with standard confidence intervals, confidence bands convey the same information as null hypothesis tests of simple slopes and/or regions of significance. Specifically, the points where the confidence bands cross zero are the boundaries of the regions of significance and so also indicate which simple slopes are significant or not significant. More important, the confidence bands also convey our certainty in our simple slope estimates and how that certainty changes as we progress across the range of our moderating variable. Typically, we will have much greater precision of estimation for medium values of the moderator, with increasing imprecision as we move to the extreme ends of the scale. Analytically, this is true because the confidence bands expand hyperbolically around the conditional effect estimate.

Evaluating Interactions Between Exogenous Predictors of the Latent Curve Factors

We just described how a main effect predictor of a latent slope factor can be conceptualized as a two-way interaction between the predictor and time. Given this, then a two-way interaction among exogenous predictors of a slope factor can in turn be conceptualized as a three-way interaction with time and should be treated as such. If we designate the first predictor as x_{1i}, the second predictor as x_{2i}, and the multiplicative interaction as $x_{1i}x_{2i}$, then intercept and linear slope equations are given as

$$\eta_{\alpha_i} = \mu_\alpha + \gamma_1 x_{1i} + \gamma_2 x_{2i} + \gamma_3 x_{1i}x_{2i} + \zeta_{\alpha_i},$$

$$\eta_{\beta_i} = \mu_\beta + \gamma_4 x_{1i} + \gamma_5 x_{2i} + \gamma_6 x_{1i}x_{2i} + \zeta_{\beta_i}. \quad (36)$$

Further, we can write the reduced form for the conditional mean of y_t as a function of x_1 and x_2 as

$$\mu_{y_t}|_{x_1,x_2} = (\mu_\alpha + \gamma_1 x_1 + \gamma_2 x_2 + \gamma_3 x_1 x_2) + (\mu_\beta + \gamma_4 x_1 + \gamma_5 x_2 + \gamma_6 x_1 x_2)\lambda_t. \quad (37)$$

We have arranged this equation in the same form as Equation 19 to illustrate how the intercepts and slopes of the simple trajectories (the first and second terms above) depend on the two predictors and their interaction. Further, this highlights that the interaction x_1x_2 itself interacts with time λ_t (via γ_6). We could equivalently rearrange the equation to show how the effect of x_1 depends on the interaction of x_2 and λ_t or how the effect of x_2 depends on the interaction of x_1 and λ_t. We now show how to compute and conduct tests of the simple slopes, followed by a discussion of the use of regions of significance and confidence bands with higher order interactions.

Estimating and Testing Simple Slopes

Unlike computing the simple slopes of main effect predictors, when exogenous variables interact in the prediction of the trajectory factors, we must evaluate the time-specific effect of one predictor at various levels of the other. Specifically, the simple slope for x_1, defined as the effect of x_1 on y at a specific point in time λ_t and a specific level of x_2, can be expressed as

$$\hat{\gamma}_1|_{x_2,\lambda_t} = \hat{\gamma}_1 + \hat{\gamma}_3 x_2 + \hat{\gamma}_4 \lambda_t + \hat{\gamma}_6 x_2 \lambda_t. \quad (38)$$

To probe this conditional effect, we must then calculate and test the effect of x_1 at various points in time within our observational window and, if x_2 is categorical, for each group defined by x_2, or, if x_2 is continuous, for high, medium, and low values of x_2. The standard errors needed to test the simple slopes are obtained by covariance algebra in much the same way as before. Because the expression for the standard error involves many terms, we do not present it here (see the Appendix for greater detail). Rather, we simply note that we could again obtain these standard errors by recoding x_2 and λ_t to place their origins at the desired values and then reestimating the model as described above for two-way interactions. The only additional complication is that to probe higher order interactions, more models have to be estimated to evaluate each possible combination of selected values for x_2 and λ_t. With these standard errors in hand, the critical ratios of each estimate to its standard error can be formed and compared with a z distribution to obtain tests of statistical significance for the simple slopes. Parallel procedures would be used to evaluate the simple slopes of x_2 for specific values of x_1 and λ_t.

Although tests of the simple slopes of x_1 and x_2 can be highly informative, our primary interest is likely to be in the simple trajectories defined at various levels of the exogenous predictors. In this case, we would obtain point estimates for the intercepts and slopes of the simple trajectories by selecting values of x_1 and x_2 and then computing the conditional intercept and slope at those values:

$$\hat{\mu}_\alpha|_{x_1,x_2} = \hat{\mu}_\alpha + \hat{\gamma}_1 x_1 + \hat{\gamma}_2 x_2 + \hat{\gamma}_3 x_1 x_2,$$

$$\hat{\mu}_\beta|_{x_1,x_2} = \hat{\mu}_\beta + \hat{\gamma}_4 x_1 + \hat{\gamma}_5 x_2 + \hat{\gamma}_6 x_1 x_2. \quad (39)$$

The standard errors again involve many terms so are not presented here (see the Appendix for greater detail). The significance of these estimates can be tested in the same way as the two-way interaction model, with the exception that we would now need to evaluate the simple trajectories at different levels of x_1 and x_2. If both predictors were continuous, and we selected high, medium, and low values of each to evaluate the simple trajectories, crossing these values would result in nine simple trajectories. If one or both were categorical, we would evaluate the simple trajectory for each group present in the analysis at selected levels of the other predictor. Finally, we could recode the predictors to place their origins at the selected values and reestimate the model to obtain the same point estimates, standard errors, and significance tests afforded by the equations.

Regions of Significance and Confidence Bands

Similar to the computation of simple slopes, the use of regions of significance and confidence bands relies on the more complex standard error expressions for conditional effects. However, unlike the computation of simple slopes, regions of significance and confidence bands are most useful when examined on a single dimension, that is, where the estimate and standard error vary as a function of a single variable at a time as was the case with main effect predictors of the trajectory parameters.[5] When interactions between exogenous predictors are added to the model, the two variables cannot be examined in isolation with respect to one another, and thus both dimensions have to be considered simultaneously. Regions of significance would be two-dimensional regions on the plane defined by the two interacting predictors, and confidence bands would evolve into confidence surfaces. This additional complexity quickly diminishes the appeal and interpretability of these procedures.

To overcome this difficulty, we propose combining these techniques with the simple slopes approach of selecting specific values of one predictor at which to evaluate the effect of the other. Thus, for instance, to evaluate the conditional effect of x_1 over time at a specific level of x_2, we could rewrite Equation 38 as

$$\hat{\gamma}_1|_{x_2,\lambda_t} = (\hat{\gamma}_1 + \hat{\gamma}_3 x_2) + (\hat{\gamma}_4 + \hat{\gamma}_6 x_2)\lambda_t. \qquad (40)$$

Note that this equation is now parallel in form to Equation 20, where the conditional effect has both an intercept (the first parenthetical term) and a slope (the second parenthetical term). Thus, if we choose specific values for x_2 at which to evaluate this relationship (i.e., $x_{2\,\text{high}}$, $x_{2\,\text{medium}}$, and $x_{2\,\text{low}}$), the conditional effect of x_1 is a linear function of λ_t, just as in Equation 20. We can compute the variances and covariances of the point estimates for the intercepts and slopes in Equation 40 using the following expressions (see the Appendix for greater detail):

$$VAR(\hat{\gamma}_1 + \hat{\gamma}_3 x_2) = VAR(\hat{\gamma}_1) + 2x_2 COV(\hat{\gamma}_1, \hat{\gamma}_3) + x_2^2 VAR(\hat{\gamma}_3),$$

$$VAR(\hat{\gamma}_4 + \hat{\gamma}_6 x_2) = VAR(\hat{\gamma}_4) + 2x_2 COV(\hat{\gamma}_4, \hat{\gamma}_6) + x_2^2 VAR(\hat{\gamma}_6),$$

$$COV(\hat{\gamma}_1 + \hat{\gamma}_3 x_2, \hat{\gamma}_4 + \hat{\gamma}_6 x_2) = COV(\hat{\gamma}_1, \hat{\gamma}_4) + x_2 COV(\hat{\gamma}_3, \hat{\gamma}_4) + x_2 COV(\hat{\gamma}_1, \hat{\gamma}_6) + x_2^2 COV(\hat{\gamma}_3, \hat{\gamma}_6). \qquad (41)$$

We now have all the information we need to compute regions of significance and plot confidence bands for the effect of x_1 over time at each selected level of x_2 using Equations 28 and 34. We would simply substitute the values computed in Equations 40 and 41 for their corresponding terms in Equations 28 and 34. Alternatively, we could avoid these calculations altogether by recoding x_2 so that the origin is at the desired level and reestimating the model (e.g., for the centered predictor x_2 reestimating the model with $x_{2\,\text{high}} = x_2 - SD_{x_2}$ to probe the effect of x_1 as a function of time at 1 standard deviation above the mean). The point estimates for $\hat{\gamma}_1$ and $\hat{\gamma}_4$ and their estimated variances and covariance can then be used directly in Equations 28 and 34. We could, of course, equivalently examine regions of significance and confidence bands for the effect of x_2 over time at each of the three selected levels of x_1. Or, if either predictor is categorical, we could examine regions of significance and confidence bands within each group.

The last possibility is to examine regions of significance and confidence bands for the intercepts and slopes of the simple trajectories. Here again we would recommend computing these within levels of one of the two predictors. For instance, choosing a specific value for x_2 at which to evaluate the influence of x_1 on the slope of the simple trajectories, we would write

$$\hat{\mu}_\beta|_{x_1,x_2} = (\hat{\mu}_\beta + \hat{\gamma}_5 x_2) + (\hat{\gamma}_4 + \hat{\gamma}_6 x_2)x_1, \qquad (42)$$

which is again a linear expression of x_1. To compute the regions of significance and confidence bands, we then need only the variances and covariance of the intercept (first parenthetical term) and slope (second parenthetical term) of this expression, given as

[5] This is true even if there are multiple main effect predictors, as the conditional effects can still be examined one predictor at a time, where the interest is in the unique effect of that predictor controlling for other predictors in the model.

$$VAR(\hat{\mu}_\beta + \hat{\gamma}_5 x_2) = VAR(\hat{\mu}_\beta) + 2x_2 COV(\hat{\mu}_\beta, \hat{\gamma}_5) + x_2^2 VAR(\hat{\gamma}_5),$$

$$VAR(\hat{\gamma}_4 + \hat{\gamma}_6 x_2) = VAR(\hat{\gamma}_4) + 2x_2 COV(\hat{\gamma}_4, \hat{\gamma}_6) + x_2^2 VAR(\hat{\gamma}_6),$$

$$COV(\hat{\mu}_\beta + \hat{\gamma}_5 x_2, \hat{\gamma}_4 + \hat{\gamma}_6 x_2) = COV(\hat{\mu}_\beta, \hat{\gamma}_4) + x_2 COV(\hat{\gamma}_5, \hat{\gamma}_4) + x_2 COV(\hat{\mu}_\beta, \hat{\gamma}_6) + x_2^2 COV(\hat{\gamma}_5, \hat{\gamma}_6). \quad (43)$$

Similar derivations could be performed to compute regions of significance and confidence bands for the simple intercepts or to examine the conditional effect of x_2 at specified levels of x_1.

Multiple-Group LCA

There are situations in which we would like to test interactions between a continuous or categorical predictor and a categorical variable denoting membership in two or more discrete groups. An implicit assumption that we have made thus far is that, although the magnitude of the relation between an exogenous predictor and the trajectory factors may vary as a function of another predictor, all other model parameters are invariant across group membership. Although often tenable, in some situations this may not be a reasonable assumption. For example, it might be expected a priori that individual trajectories will differ in form and function across a treatment and a control group; across males and females; or across Blacks, Hispanics, and Caucasians (see, e.g., McArdle, 1989; Muthén, 1989; Muthén & Curran, 1997). Instead of assuming invariance of model parameters across groups, we may instead want to empirically evaluate the validity of this assumption. Multiple-groups SEM is a powerful analytic tool that allows for the estimation and testing of model invariance across two or more discrete groupings, and we can make use of this approach here.

Briefly, the multiple-group SEM is based on the same model equations as described earlier, but these parameters have the potential to vary in magnitude from one group to another. The fundamental latent curve equations can thus be expressed as

$$\mathbf{y}^g = \mathbf{\Lambda}^g \boldsymbol{\eta}^g + \boldsymbol{\varepsilon}^g, \quad (44)$$

$$\boldsymbol{\eta}^g = \boldsymbol{\mu}_\eta^g + \mathbf{\Gamma}^g \mathbf{x} + \boldsymbol{\zeta}^g, \quad (45)$$

where $g = 1, 2, \ldots, G$ represents membership in one of G discrete groups. Given this, the group-specific covariance and mean structure are defined as

$$\mathbf{\Sigma}(\boldsymbol{\theta})^g = \mathbf{\Lambda}^g(\mathbf{\Gamma}^g \mathbf{\Phi}^g \mathbf{\Gamma}^{g\prime} + \mathbf{\Psi}^g)\mathbf{\Lambda}^{g\prime} + \mathbf{\Theta}_\varepsilon^g, \quad (46)$$

$$\boldsymbol{\mu}(\boldsymbol{\theta})^g = (\mathbf{\Lambda}^g \boldsymbol{\mu}_\eta^g + \mathbf{\Lambda}^g \mathbf{\Gamma}^g \boldsymbol{\mu}_x^g), \quad (47)$$

highlighting that all model parameters and model-implied moment structures can potentially vary across groups. If all matrices are defined to be invariant over groups, then the results simplify to a standard single-group analysis of the pooled sample. If all matrices are defined to vary over groups, then the results correspond to a standard single-group analysis estimated within each group separately. Typically, some combination of variant and invariant matrices are defined such that some parts of the model are equal over groups and other parts are not (see, e.g., Bollen, 1989).

We can make use of this model formulation to test a subset of the interactions described earlier. Specifically, we can consider interactions between a discrete categorical variable with G levels and any other type of exogenous variable (categorical or continuous). For example, consider the conditional LCA with a single dichotomous predictor in Equation 26. This could be equivalently estimated as a two-group SEM with equality constraints imposed on all parameters except the latent factor means. The difference between the two intercept factor means and between the two slope factor means is equal to $\hat{\gamma}_1$ and $\hat{\gamma}_2$ from Equation 26, respectively. However, the two-group framework in addition allows us to test the invariance assumptions by estimating a series of nested models in which equality constraints are removed and evaluated. If there are meaningful differences in the covariance structures of the errors or the latent factors across the two groups, these can be incorporated into the model prior to evaluating group differences in factor means.

We could further extend this two-group framework to test the interaction between discrete group membership and one or more continuous or categorical predictors. For example, consider the conditional LCA with a two-way interaction presented in Equation 36 in which x_1 is dichotomous and x_2 is continuous. This model could be equivalently estimated by regressing the intercept and slope factors on the single continuous predictor x_2 in the two-group LCA where group is defined by x_1. A significant difference between the factor means across group reflects a main effect of x_1, a significant regression of the growth factors on x_2 (where the magnitude of this effect is held equal over groups) reflects a main effect of x_2, and a significant difference in the magnitude of the regression estimates between the two groups (assessed when the equality constraint on the regression coefficients is removed) would imply the presence of an interaction between x_1 and x_2. Again, equality constraints could be used to empirically test the appropriateness of assuming equal parameters over group. Depending on the outcomes of these tests, group differences in model parameters could be incorporated prior to testing the relations between the exogenous predictors and the latent growth factors within each group.

In one sense this approach is limited in that testing interactions between nondiscretely distributed covariates are not currently possible using this framework (e.g., a continuous by continuous interaction). However, in another sense this is a particularly powerful approach given that not only can we explicitly test for mean differences as a function of group membership (as we did before), but we can also explicitly test differences in variances and covariances of residuals ($\mathbf{\Theta}_{\varepsilon}^{g}$), latent curve factors ($\mathbf{\Psi}^{g}$), or exogenous predictors ($\mathbf{\Phi}^{g}$). Thus, we need not assume that these covariance structures are invariant over group but can explicitly estimate and test for these differences. This may be particularly salient here given the role of these parameter estimates in the computation of standard errors for simple trajectories. Given space constraints, we do not pursue this powerful approach in greater detail here, but see Curran and Muthén (1999), McArdle (1989), and Muthén and Curran (1997) for further discussion.

We now conclude with an empirical example in which we test a categorical by continuous interaction in an unbalanced latent curve model with missing data over time using the set of techniques described above.

Empirical Example

Data were drawn from the National Longitudinal Study of Youth; specific details of sample selection and measures were presented in Curran (1997).[6] Selection criteria resulted in a subsample of children ($n = 405$) ranging in age from 6 to 8 years at Time 1 (49% female). Children were interviewed at least once and up to four times at approximately 24-month intervals. All 405 children were interviewed at Wave 1, 374 were interviewed at Wave 2, 297 were interviewed at Wave 3, 294 were interviewed at Wave 4, and 221 were interviewed at all four assessment waves. Given the age variability at Wave 1, the approximate 2-year spans between assessments, and the participant attrition over time, this can be considered an unbalanced design with missing data.[7] To simultaneously model the complex characteristics of this data while retaining all observed cases, we used a cohort-sequential design (e.g., Miyazaki & Raudenbush, 2000) with raw maximum-likelihood estimation (e.g., Neale, Boker, Xie, & Maes, 2002; Wothke, 2000). This allowed for the incorporation of both missing data and individually varying time between assessments.

There were three measures of interest for the current example. At Wave 1, the gender of the child was assessed and coded; a value of 0 denoted female and a value of 1 denoted male. A measure of emotional support of the child in the home was assessed at Wave 1, and this was a continuous measure that ranged from 0 to 13, where higher values reflected higher levels of emotional support of the child at home; emotional support was centered about the mean. Finally, up to four repeated measures of antisocial behavior in the child were assessed; this was a continuous measure representing the sum of six items assessing child antisocial behavior over the previous 3 months. Values ranged from 0 to 12, where higher values reflected higher levels of child antisocial behavior; antisocial behavior was not centered about the mean.

There were three motivating questions of interest. First, what is the optimal functional form of the mean developmental trajectory of antisocial behavior over time? Second, is there evidence for meaningful individual variability in trajectories around these mean values? Finally, is there an interaction between gender and emotional support in the prediction of the trajectories such that the magnitude of the relation between emotional support varies for boys and girls? The conditional cohort-sequential latent curve model is presented in Figure 2. It can be seen that the raw maximum-likelihood estimator is significantly advantageous in that although no single child provided more than four repeated measures, we can estimate a trajectory spanning 10 years of development. The factor loadings are coded as $\lambda_{it} = age_{it} - 6$, where 6 was the youngest child in the sample at the first assessment and age was rounded to the nearest year; this allows for the intercept to be defined as the model-implied value of antisocial behavior for the youngest age in the sample.

We first estimated an unconditional latent curve model (thus omitting the three covariates shown in Figure 2) to establish the optimal growth function over time.[8] We began by estimating a linear model with all time-specific residual variances set to be equal over time. Because of the unbalanced nature of the data (and, more specifically, the zero covariance coverage for some elements of the sample covariance matrix precluding the estimation of a saturated model), there are no standard stand-alone chi-square tests or incremental fit indices available, just as there are not in traditional HLMs. However, we can obtain a chi-square difference test between two nested models. Using this method, we first tested the adequacy of the equal error variances over time and found that there was not a significant decrement in model fit associated with this restriction,

[6] All raw data and computer code are available on the Web for download (http://www.unc.edu/~curran).

[7] There are important assumptions made about the mechanism that gave rise to the missing data, but a full discussion of this is beyond the scope of the current article. Please see Allison (2001), Arbuckle (1996), and Wothke (2000) for excellent discussions of this topic.

[8] Although we estimated all of our latent curve models in Amos (Arbuckle, 1999), these analyses can be fully replicated with any SEM software package that allows for full-information maximum-likelihood estimation with missing data.

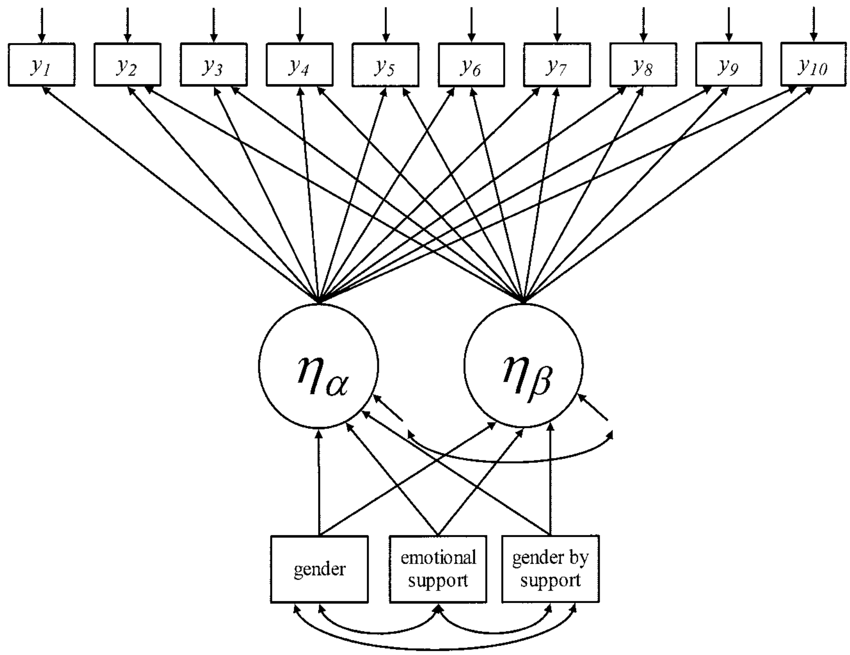

Figure 2. Path diagram of 10 time-point cohort-sequential conditional linear latent curve model regressed on the main effect of gender, the main effect of emotion, and the interaction between gender and emotion; η_α and η_β represent the latent intercept and latent slope of the trajectory, respectively.

and we retained the equal residuals over time.[9] We next tested the improvement in model fit with the inclusion of a quadratic latent curve factor to capture potential nonlinearity over time, and this too did not result in a significant improvement in model fit and was thus not retained.

To evaluate the overall fit of the final model, we considered the magnitude of the standardized parameter estimates and squared multiple correlations, the presence of large and significant modification indices, and the magnitude and distribution of residuals between the observed and model-implied covariance and mean structures. The stability of the model was also checked by examining the number of iterations needed to converge, the sensitivity of the solution to variations in start values, and the potential impact of influential observations. Space constraints preclude a detailed presentation of all of this information, but all of these results suggested that there was an excellent and stable fit of the linear latent curve model with equal residual variances over time to the observed data.

We then regressed the latent intercept and linear slope factors on the main effects of gender and emotional support and the multiplicative interaction between these two measures. Evaluating the same criteria as described above, there was again an excellent fit of the hypothesized model to the observed data. The parameter estimates and standard errors for this model are presented in Table 1. Of greatest importance to our discussion here is the significant regression of the latent slope factor on the multiplicative interaction between gender and emotional support ($\hat{\gamma} = -.029$, $p = .05$). This implies that the strength of the relation between emotional support and developmental trajectories of antisocial behavior varies as a function of child gender. However, as we highlighted above, given the parameterization of time in the factor loading matrix, we must explicitly probe this two-way interaction between exogenous variables as a three-way interaction with time. To accomplish this, we used the methods described above to calculate the point estimates and standard errors for the model-implied trajectories of antisocial behavior at low, medium, and high levels of emotional support within boys and girls (as in Equation 39; see Figure 3).

First, within boys (see the top panel of Figure 3), although all three simple trajectories are increasing over time, the magnitude of increase is significantly larger with decreasing levels of emotional support in the home. Of importance, the

[9] We identified a Heywood case for the residual variance at the final time point. However, this was small in magnitude and the Wald test was nonsignificant, suggesting this was not due to model misspecification (Chen, Bollen, Paxton, Curran, & Kirby, 2001). Further, there were no Heywood cases with the equality constraints imposed, and this imposition did not lead to decrement in model fit and was thus retained.

Table 1
Parameter Estimates and Standard Errors From the Main Effects and Two-Way Interaction Predicting the Intercept and Slope Factors

Predictor variable	Intercept factor Parameter	Intercept factor *SE*	Slope factor Parameter	Slope factor *SE*
Child gender	0.829	.161	0.013	.035
Time 1 emotional support	−0.194	.048	0.012	.010
Gender by support interaction	0.044	.070	−0.029	.015
Intercept term of the prediction equation	1.217	.114	0.066	.024

Note. Model results are based on $N = 405$ assessed at Time 1, but sample size varied at Times 2, 3, and 4.

slopes of the simple trajectories of antisocial behavior are significantly positive at low and medium levels of support but are not significantly different from zero at high support. In comparison, whereas the simple trajectories are diverging for boys over time, they are converging for girls (see the bottom panel of Figure 3). The ranking of the simple trajectories is similar over gender (e.g., low emotional support is associated with the greatest antisocial behavior over time); however, the simple trajectories are significantly increasing for girls at medium and high levels of support but are not significantly different from zero at low levels of support. Note that these complex conditional relations are not apparent without further probing of the interactive effect.

The above tests show that the simple trajectories are significantly increasing over time at some values of emotional support, but not at others. To identify the exact values of emotional support at which the slope of the simple trajectory moves from nonsignificant to significant, we calculated the regions of significance and plotted the associated confidence bands (see Figure 4). The nonshaded area in the figure reflects the range of emotional support for which the slopes of the simple trajectories were significantly different from zero, and the shaded area reflects the range for which the simple trajectories were not. It can be seen that for boys, the simple trajectories are significantly positive between −15.6 and 1.42 units on the scale of support, but not outside

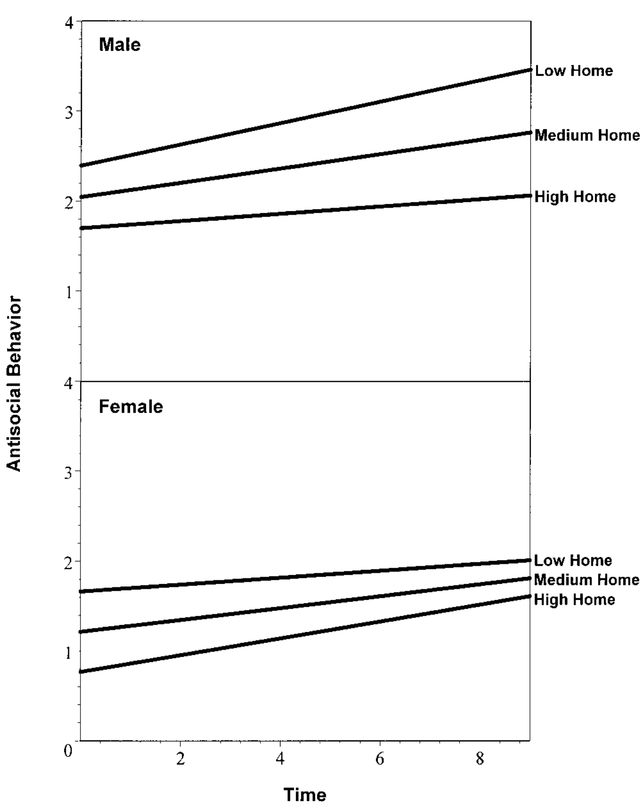

Figure 3. Simple trajectories of antisocial behavior development for male and female adolescents plotted as a function of home support (home) with trajectories shown at the mean of home support (medium) and at the mean ±1 standard deviation of home support (high and low).

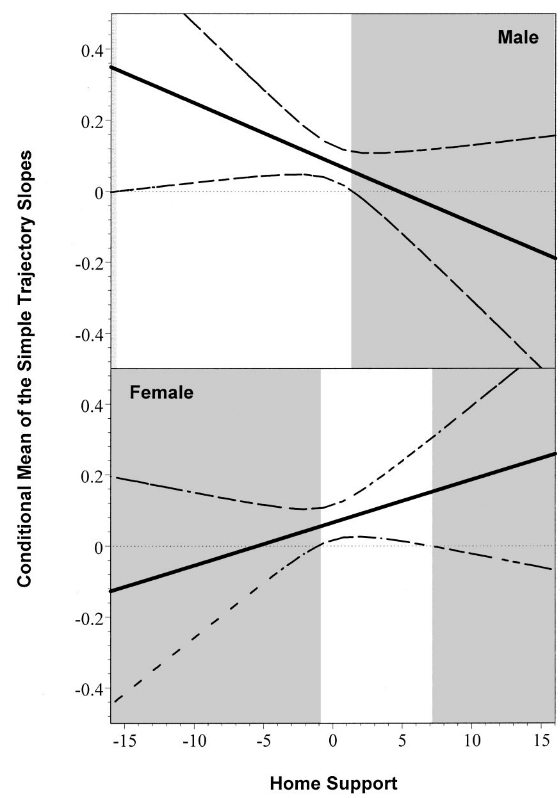

Figure 4. The conditional mean of the simple trajectory slopes as a function of home support by gender. Dashed lines represent nonsimultaneous 95% confidence bands for the function. The points at which the confidence bands cross zero demarcate the nonsimultaneous regions of significance. Regions of home support over which the slope parameter is significantly different from zero are nonshaded, whereas regions over which the slope parameter is nonsignificant are shaded.

of this range. Given support is centered around the mean, this reflects that simple slopes are not significant at 1.42 units (or 0.61 *SD*) above the mean. In contrast, for girls, the simple trajectories are significantly positive between −1.15 and 7.04 units on the scale of support, but not outside of this range. This implies that slopes are nonsignificant when support values are less than −1.1 (or 0.50 *SD*) below the mean or when support values are greater than 7.0 (or 3.05 *SD*) above the mean. Note that the overlap of nonshaded area between boys and girls (i.e., between −1.15 and 1.42) suggests the range of emotional support in which simple trajectories operate similarly across gender (although we do not formally test this here).

In Figure 4 we plotted the relation between emotional support (on the *x*-axis) and the model-implied value of the slope of the simple trajectory (on the *y*-axis). In contrast, we can alternatively plot the relation between time (on the *x*-axis) and the magnitude of the effect of home support on antisocial behavior (on the *y*-axis) and compute regions of significance and confidence bands for this effect (as in Equation 40; see Figure 5).[10] It can be seen that emotional support exerts a significant negative influence on antisocial behavior in boys across all ages under study. In contrast, emotional support similarly exerts a significant negative influence on antisocial behavior in girls, but only up to about age 13 (recall that time was coded as child age minus 6); after this age, there is not a significant relation between support and antisocial behavior in girls. This helps clarify the differential relation between simple trajectories of antisocial behavior and emotional support for boys and girls presented in Figure 3. More specifically, emotional support is consistently negatively associated with antisocial behavior, but the magnitude of this effect strengthens with increasing age for boys but weakens with increasing age for girls. Only by calculating the regions of significance can we identify these complex conditional relations over time.

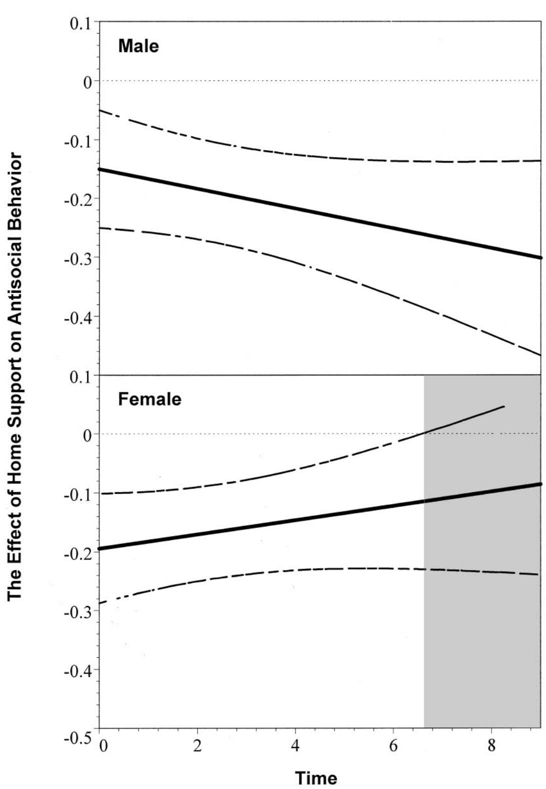

Figure 5. The effect of home support as a function of time by gender. Dashed lines represent nonsimultaneous 95% confidence bands for the function. The points at which the confidence bands cross zero demarcate the nonsimultaneous regions of significance. Regions of time over which the effect of home support is significantly different from zero are nonshaded, whereas regions over which the effect of home support is nonsignificant are shaded.

The above models were all estimated pooling boys and girls into a single group and estimating gender differences by including a dummy-coded covariate to denote gender. However, as we noted earlier, this imposes a strong assumption that the covariance structures of the random growth processes and time-specific residuals are invariant over gender. This assumption is not testable in the single group model but can be explicitly evaluated using a two-group latent growth model. We estimated a series of two-group latent curve models in which a variety of equality constraints were imposed both within and across gender. Given space constraints, we do not fully explicate these results here. However, the empirical results indicated that there was no evidence of meaningful gender differences in the covariance structure of the random growth or residual model components. We thus concluded that it was appropriate to test these models by pooling data from boys and girls and assuming other model parameters were invariant over gender.

Limitations and Future Directions

Although we have discussed a number of potentially useful conditions under which these techniques can be applied, there are several topics that we did not explore in greater detail. First, we did not detail the case in which an exogenous variable interacts with itself such that there is a curvilinear relation between the exogenous variable and the trajectory parameters. However, the methods we have described here easily generalize to include such curvilinear

[10] Note that there is a critical difference between Figures 3 and 5. Although both figures denote time on the *x*-axis, in Figure 3 the *y*-axis represents antisocial behavior, whereas in Figure 5 the *y*-axis represents the magnitude of the effect of support in the prediction of antisocial behavior. In other words, Figure 3 represents the simple trajectories of antisocial behavior over time at three specific levels of support as a function of gender, and Figure 5 represents the magnitude of the relation between support and antisocial behavior over time as a function of gender.

interactions in LCA. Second, although we only considered a linear latent curve model here, our proposed methods would directly apply to higher order polynomial functions as well. For example, all interactions would be one order higher if a quadratic growth factor were estimated, given that this function is in part defined by the interaction of time with itself. Third, an important assumption underlying all of the models that we have presented here is that the exogenous manifest variables are fixed and measured without error (e.g., Bollen, 1989). It is well-known that measurement error in the exogenous predictors can serve to both bias parameter estimates and decrease statistical power. Multiple indicator latent factors could be used to explicitly model measurement error, although additional complexities arise when one is estimating interactions among latent variables (e.g., Bollen, 1996; Li, Duncan, & Acock, 2000). However, the methods that we have described for probing interactions can all be applied to these more complex measurement models as well.

One direction for future research is to further explore the correspondence between multiplicative interactions of dichotomous and continuous predictors with a multiple-group SEM approach. Although we briefly addressed this issue here, much interesting work is yet to be done in extending the above methods in a multiple-group framework to allow for explicit tests of heterogeneity in covariance structures prior to testing group differences in conditional means (for examples of this approach, see Curran & Muthén, 1999; Muthén & Curran, 1997). Further, as we noted earlier, we have only focused on the interactions between two or more exogenous variables in the prediction of the latent growth factors. New methods are becoming increasingly available to estimate interactions among the latent factors themselves, and these techniques could be applied in interesting ways in the latent curve model (see, e.g., Li et al., 2000, 2001). Finally, in all of our methods described here it is assumed that the factor loadings are fixed to specific values and not estimated from the data. There are several powerful variants of the latent curve model that include one or more loadings that are freely estimated from the data and thus have associated standard errors (e.g., Browne, 1993; du Toit & Cudeck, 2001; McArdle, 1989); future developments could extend our methods to account for these estimated loadings and standard errors.

Conclusion

Our goal here was to analytically develop and empirically demonstrate methods for testing and probing main effects and interactions in LCA. We have argued that it is useful to reconceptualize the indirect effect of an exogenous predictor on the repeated measures as mediated via the factor loading matrix as an implicit multiplicative interaction between the predictor variable and time. We have analytically demonstrated that classic techniques for testing interactions can be generalized to a broad class of conditional latent curve models. Further, we have empirically demonstrated that these methods allow for the extraction of much greater information from the model than is typically considered. The methods described here not only provide a mechanism for more fully understanding the influence of main effect predictors of the latent curve factors but also directly extend to the testing and probing of higher order interactions. We believe that these methods can be exploited to gain a clearer and more comprehensive understanding of potentially complex model results, and we recommend these methods be considered whenever predictors are tested within the latent curve framework.

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Appendix

Technical Details

Derivation of Standard Errors for Conditional Effects

Here we provide the derivation of the effect estimates and standard errors for the conditional effect of one predictor at specified levels of another. These standard errors are critical for conducting tests of simple slopes, computing regions of significance, and constructing confidence bands. Each procedure relies on the general formula for calculating the variance of a linear composite of random variables (see Aiken & West, 1991, pp. 25–26; Morrison, 1990, p. 83). SAS code and a Web-based interface can be accessed at http://www.unc.edu/~curran that automate the calculation of all of the results described below.

Asymptotic Variance of a Linear Composite

Consider the general linear composite

$$v = a_1 z_1 + \cdots + a_q z_q, \quad \text{(A1)}$$

where $z_1, z_2, \ldots, z_q$ are normally distributed random variables and $a_1, a_2, \ldots, a_q$ are the fixed coefficients associated with each variable. Note that we may view all of the conditional effect estimates described in this article as linear composites of other estimated parameters (i.e., Equations 20, 23, 38, and 39, as well as the terms within Equations 40 and 42). In each case, the fixed coefficients are the selected (or fixed) values of the predictors that we use to evaluate the conditional effects, and the normally distributed random variables are the parameter estimates for the model. The parameter estimates are viewed as random variables because they would be expected to change from one replication to the next, varying about the true population value. The assumption of normality for the sampling distribution of these estimates is assured asymptotically by the properties of the maximum-likelihood estimator.

To further evaluate this conditional effect, we must estimate the variance of its sampling distribution and take the square root of this quantity, the standard error. Although we cannot directly measure the standard error of the conditional effect, we can estimate this value. To do so, we make use of a general property of linear composites, that the variance of a linear composite of normally distributed random variables is

$$VAR(v) = VAR(\mathbf{a'Z}) = \mathbf{a'Ca}, \quad \text{(A2)}$$

where $\mathbf{a}$ is a $q \times 1$ column vector containing the $a_1, a_2, \ldots, a_q$ fixed coefficients and $\mathbf{C}$ is the $q \times q$ covariance matrix for $z_1, z_2, \ldots, z_q$. To use this formula, we simply insert into the $\mathbf{a}$ vector specific values for the predictors and insert into $\mathbf{C}$ the estimated asymptotic covariance matrix of the estimated effects of these predictors.

To provide a concrete example, consider Equation 20,

$$\hat{\gamma}_1|_{\lambda_t} = \hat{\gamma}_1 + \hat{\gamma}_2\lambda_t, \quad \text{(A3)}$$

which gives the conditional effect (simple slope) of the predictor x on the repeated measures at a given point in time λ_t. Viewing the specified value λ_t as fixed, and the quantities of $\hat{\gamma}_1$ and $\hat{\gamma}_2$ as random variables with a bivariate normal sampling distribution, we can compute the variance of $\hat{\gamma}_1|_{\lambda_t}$ by defining

$$\mathbf{a'} = [1 \quad \lambda_t],$$

$$\mathbf{C} = \begin{bmatrix} VAR(\hat{\gamma}_1) & COV(\hat{\gamma}_1, \hat{\gamma}_2) \\ COV(\hat{\gamma}_1, \hat{\gamma}_2) & VAR(\hat{\gamma}_2) \end{bmatrix}, \quad \text{(A4)}$$

where $VAR(\hat{\gamma}_1)$, $VAR(\hat{\gamma}_2)$, and $COV(\hat{\gamma}_1, \hat{\gamma}_2)$ are obtained from the asymptotic covariance matrix of the estimates (the inverse of the information matrix), which can be output from most SEM software packages. The variance for $\hat{\gamma}_1|_{\lambda_t}$ can be derived by applying Equation A2:

$$VAR(\hat{\gamma}_1|_{\lambda_t}) = [1 \quad \lambda_t]\begin{bmatrix} VAR(\hat{\gamma}_1) & COV(\hat{\gamma}_1, \hat{\gamma}_2) \\ COV(\hat{\gamma}_1, \hat{\gamma}_2) & VAR(\hat{\gamma}_2) \end{bmatrix}\begin{bmatrix} 1 \\ \lambda_t \end{bmatrix},$$

$$= VAR(\hat{\gamma}_1) + 2\lambda_t COV(\hat{\gamma}_1, \hat{\gamma}_2) + \lambda_t^2 VAR(\hat{\gamma}_2). \quad \text{(A5)}$$

The standard error for $\hat{\gamma}_1|_{\lambda_t}$ is then simply the square root of this value, or $SE(\hat{\gamma}_1|_{\lambda_t}) = VAR(\hat{\gamma}_1|_{\lambda_t})^{1/2}$.

Identical computations can be performed to obtain the variances for the other conditional effects described in this article (i.e., the intercepts and slopes of the simple trajectories in Equation 23). This is true even for conditional effect estimates that involve interactions among predictors (i.e., Equations 38 and 39). For instance, if we take Equation 38,

$$\hat{\gamma}_1|_{x_2,\lambda_t} = \hat{\gamma}_1 + \hat{\gamma}_3 x_2 + \hat{\gamma}_4\lambda_t + \hat{\gamma}_6 x_2\lambda_t, \quad \text{(A6)}$$

we can write

$$\mathbf{a'} = [1 \quad x_2 \quad \lambda_t \quad x_2\lambda_t],$$

$$\mathbf{C} = \begin{bmatrix} VAR(\hat{\gamma}_1) & COV(\hat{\gamma}_1, \hat{\gamma}_3) & COV(\hat{\gamma}_1, \hat{\gamma}_4) & COV(\hat{\gamma}_1, \hat{\gamma}_6) \\ COV(\hat{\gamma}_1, \hat{\gamma}_3) & VAR(\hat{\gamma}_3) & COV(\hat{\gamma}_3, \hat{\gamma}_4) & COV(\hat{\gamma}_3, \hat{\gamma}_6) \\ COV(\hat{\gamma}_1, \hat{\gamma}_4) & COV(\hat{\gamma}_3, \hat{\gamma}_4) & VAR(\hat{\gamma}_4) & COV(\hat{\gamma}_4, \hat{\gamma}_6) \\ COV(\hat{\gamma}_1, \hat{\gamma}_6) & COV(\hat{\gamma}_3, \hat{\gamma}_6) & COV(\hat{\gamma}_4, \hat{\gamma}_6) & VAR(\hat{\gamma}_6) \end{bmatrix}. \quad \text{(A7)}$$

and apply Equation A2 to get the corresponding variance estimate for $\hat{\gamma}_1|_{x_2,\lambda_t}$. The variance of any other linear composite of estimated model parameters can be estimated similarly. For instance, though we do not present this here, Equation A2 can be used to compute variance estimates for conditional effects in nonlinear latent curve models where the trajectories are captured by higher order polynomial functions of time.

The variance and covariance estimates for the two terms in Equations 40 and 42 can also be viewed as linear composites, and the variances of these quantities can be estimated in the same way. However, we also need to identify the covariance of these linear composites. To do so, we simply note that the formula in Equation A2 generalizes to systems of r equations by defining $\mathbf{v}$ to be a column vector containing $v_1, v_2, \cdots, v_r$ linear composites of **Z,** and **A** to be a matrix with columns $\mathbf{a}_1, \mathbf{a}_2, \cdots, \mathbf{a}_r$ corresponding to the vector of fixed coefficients associated with each linear composite. The covariance matrix for the linear composites, denoted $\mathbf{\Sigma}_v$, can then be obtained by

$$\mathbf{\Sigma}_v = \mathbf{A'CA}, \tag{A8}$$

where **C** is again the covariance matrix of **Z.** Thus, for instance, for Equation 40 we have two linear composites of interest, $v_1 = \hat{\gamma}_1 + \hat{\gamma}_3 x_2$ and $v_2 = \hat{\gamma}_4 + \hat{\gamma}_6 x_2$. To express these in terms of the same set of parameter estimates **Z,** we can write them as

$$\begin{aligned} v_1 &= (1)(\hat{\gamma}_1) + (0)(\hat{\gamma}_4) + (x_2)(\hat{\gamma}_3) + (0)(\hat{\gamma}_6), \\ v_2 &= (0)(\hat{\gamma}_1) + (1)(\hat{\gamma}_4) + (0)(\hat{\gamma}_3) + (x_2)(\hat{\gamma}_6). \end{aligned} \tag{A9}$$

The fixed coefficients from these two equations then define the matrix **A,** and we can define **C** in the same way as before:

$$\mathbf{A'} = \begin{bmatrix} 1 & 0 & x_2 & 0 \\ 0 & 1 & 0 & x_2 \end{bmatrix},$$

$$\mathbf{C} = \begin{bmatrix} VAR(\hat{\gamma}_1) & COV(\hat{\gamma}_1, \hat{\gamma}_4) & COV(\hat{\gamma}_1, \hat{\gamma}_3) & COV(\hat{\gamma}_1, \hat{\gamma}_6) \\ COV(\hat{\gamma}_1, \hat{\gamma}_4) & VAR(\hat{\gamma}_4) & COV(\hat{\gamma}_4, \hat{\gamma}_3) & COV(\hat{\gamma}_4, \hat{\gamma}_6) \\ COV(\hat{\gamma}_1, \hat{\gamma}_3) & COV(\hat{\gamma}_4, \hat{\gamma}_3) & VAR(\hat{\gamma}_3) & COV(\hat{\gamma}_3, \hat{\gamma}_6) \\ COV(\hat{\gamma}_1, \hat{\gamma}_6) & COV(\hat{\gamma}_4, \hat{\gamma}_6) & COV(\hat{\gamma}_3, \hat{\gamma}_6) & VAR(\hat{\gamma}_6) \end{bmatrix}. \tag{A10}$$

Then solving Equation A8 will result in the variance and covariance estimates given in Equation 41. Similar procedures can be used to obtain variance and covariance estimates for the terms in Equation 42 as given in Equation 43.

Tests of Simple Slopes and Regions of Significance

In conducting tests of simple slopes, we select specific values for our moderating variable, use the formulas above to identify the effect estimate and standard error at that specified value, and then form the critical ratio of these quantities to obtain a z statistic. For a concrete example, consider Equation 22:

$$z_{\hat{\gamma}_1|\lambda_t} = \frac{\hat{\gamma}_1|_{\lambda_t}}{SE(\hat{\gamma}_1|_{\lambda_t})}. \tag{A11}$$

We can replace the numerator and denominator of the ratio with the values obtained from Equations A3 and A5, giving

$$z_{\hat{\gamma}_1|\lambda_t} = \frac{\hat{\gamma}_1 + \hat{\gamma}_2\lambda_t}{[VAR(\hat{\gamma}_1) + 2\lambda_t COV(\hat{\gamma}_1, \hat{\gamma}_2) + \lambda_t^2 VAR(\hat{\gamma}_2)]^{1/2}}. \tag{A12}$$

To compute and test simple slopes for the effect of x, we would choose a specific value for λ_t and then solve for $z_{\hat{\gamma}_1|\lambda_t}$.

To compute regions of significance, we reverse the unknown in this equation, selecting a specific critical value z_{crit} (i.e., ± 1.96 for an alpha level of .05), and then solve for the values of the moderator that yield z_{crit}. These values are on the threshold of significance, indicating the exact points on the scale of the moderating variable where the conditional effect passes from significance to nonsignificance (or vice versa). Thus, to compute these values for our example, we would replace $z_{\hat{\gamma}_1|\lambda_t}$ with the specified critical value z_{crit}:

$$z_{\text{crit}} = \frac{\hat{\gamma}_1 + \hat{\gamma}_2\lambda_t}{[VAR(\hat{\gamma}_1) + 2\lambda_t COV(\hat{\gamma}_1, \hat{\gamma}_2) + \lambda_t^2 VAR(\hat{\gamma}_2)]^{1/2}}. \tag{A13}$$

To solve this equation for the values of λ_t that yield z_{crit}, we first square both sides

$$z_{\text{crit}}^2 = \frac{(\hat{\gamma}_1 + \hat{\gamma}_2\lambda_t)^2}{VAR(\hat{\gamma}_1) + 2\lambda_t COV(\hat{\gamma}_1, \hat{\gamma}_2) + \lambda_t^2 VAR(\hat{\gamma}_2)}, \tag{A14}$$

then expand the numerator and multiply both sides by the denominator:

$$z_{\text{crit}}^2 VAR(\hat{\gamma}_1) + 2z_{\text{crit}}^2 \lambda_t COV(\hat{\gamma}_1, \hat{\gamma}_2) + z_{\text{crit}}^2 \lambda_t^2 VAR(\hat{\gamma}_2) = \hat{\gamma}_1^2 + 2\lambda_t \hat{\gamma}_1 \hat{\gamma}_2 + \lambda_t^2 \hat{\gamma}_2^2. \tag{A15}$$

Finally, we can subtract the right-hand expression from both sides of the equation and collect terms, yielding

$$[z_{\text{crit}}^2 VAR(\hat{\gamma}_2) - \hat{\gamma}_2^2]\lambda_t^2 + \{2[z_{\text{crit}}^2 COV(\hat{\gamma}_1, \hat{\gamma}_2) - \hat{\gamma}_1\hat{\gamma}_2]\}\lambda_t + [z_{\text{crit}}^2 VAR(\hat{\gamma}_1) - \hat{\gamma}_1^2] = 0 \tag{A16}$$

as given in Equation 28. As noted in the text, given this arrangement of terms, the quadratic formula can be used to solve for the two roots of λ_t that satisfy this equality, where these roots demarcate the boundaries to the regions of significance. Similar computations can be performed to obtain the regions for other conditional effects in the latent curve model. For instance, to obtain regions on x for which the intercepts and slopes of the simple trajectories are significantly different from zero, we would simply use Equation 25 in place of Equation 22.

In the case in which two exogenous variables interact in the prediction of the latent trajectory factors, the effect and standard error expressions will be considerably more complicated. This three-way interaction with time means that each conditional effect is a function of two moderating variables that themselves interact. However, as we noted within the main text, if we select specific values for one moderator, then the conditional effect formula can be reduced to a linear expression with quadratic variance just as in the simpler case considered above (i.e., Equations 40 and 42). Once variance and covariance estimates for the terms of these expressions have been computed using Equation A8, as shown above, the same algebraic steps can be followed to obtain the regions of significance.

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