Comparing Methods for Modeling Nonlinearity in Latent Curve Models

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Introduction

Why is it important to consider nonlinear models?

- In the social and behavioral sciences, we often study change over time.
- Longitudinal data analysis is often the most powerful method for answering these questions.
- **Trends over time often follow a nonlinear trajectory.**
    - The "Monitoring the Future" project (Johnston, O'Malley and Bachman (2002)

Problems engendered by ignoring nonlinearity:

- By not modeling nonlinearity, the researcher is fitting an *ipso facto* misspecified model.
- Ignoring the functional form can lead to biased inferences or limiting conclusions.

The Latent Curve Model (LCM)

The LCM model is a special case of the Confirmatory Factor Analysis (CFA) sub-model within SEM.

$$\mathbf{Y} = \Lambda_y \eta + \Theta_\varepsilon$$

where the structure of the curve is determined by the values of Λ_y.

Issues with Nonlinearity in LCM

- **Two Kinds of Nonlinearity**:
  - Nonlinearity of **form** which refers to the nonlinearity of the trajectory function.
  - Nonlinearity in the **parameters** where at least one parameter is a function of another.
- It is the second type of nonlinearity that is problematic for LCM. **In LCM, the parameters must enter the model linearly**. Thus, LCM does not transition directly from the linear to the nonlinear model.
- Nonlinearity is often modeled using polynomials, because the polynomial family of functions is linear in the parameters (additive):

$$Y_{it} = \beta_{0i} + \sum_{p=1}^{P} \beta_{pi} X_t^p + \varepsilon_{it}$$

- The polynomials are fit by assigning values to Λ (see $\Lambda_{\mathbf{Q}}$ & $\Lambda_{\mathbf{C}}$ respectively).

$$\Lambda_{\mathbf{Q}} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \\ 1 & 4 & 16 \\ 1 & 5 & 25 \end{bmatrix} \quad \Lambda_{\mathbf{C}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & 8 \\ 1 & 3 & 9 & 27 \\ 1 & 4 & 16 & 64 \\ 1 & 5 & 25 & 125 \end{bmatrix}$$

Three approaches to nonlinear modeling in LCM

The **Fully Latent** approach (McArdle 1988, 1989; Meredith & Tisak 1990):

- Allows some loadings (λ_{22} - λ_{52}) to be freely estimated (see $\Lambda_{\mathbf{FL}}$).
- Estimates a latent intercept and a latent "shape" parameter.

The **Conditionally Linear** approach (Blozis & Cudeck 1999; Cudeck 1996; du Toit & Cudeck 2001):

- Puts constraints on $\Lambda_{\mathbf{CL}}$ (shown below for the monomolecular function).
- Estimates a latent intercept and slope with a fixed growth parameter.

The **Linearization** approach (Browne 1993; Browne & du Toit 1991) :

- Puts constraints on $\Lambda_{\mathbf{TS}}$ (shown below for the monomolecular function).
- Estimates three latent factors.

$$\Lambda_{\mathbf{FL}} = \begin{bmatrix} 1 & 0 \\ 1 & \lambda_{22} \\ 1 & \lambda_{32} \\ 1 & \lambda_{42} \\ 1 & \lambda_{52} \\ 1 & 1 \end{bmatrix} \quad \Lambda_{\mathbf{CL}} = \begin{bmatrix} 1 & e^{-\beta_2 0} - 1 \\ 1 & e^{-\beta_2 1} - 1 \\ 1 & e^{-\beta_2 2} - 1 \\ 1 & e^{-\beta_2 3} - 1 \\ 1 & e^{-\beta_2 4} - 1 \\ 1 & e^{-\beta_2 5} - 1 \end{bmatrix} \quad \Lambda_{\mathbf{TS}} = \begin{bmatrix} 1 & 1-e^{-X_t\beta_2} & \beta_1 e^{-X_t\beta_2} X_t \\ 1 & 1-e^{-X_t\beta_2} & \beta_1 e^{-X_t\beta_2} X_t \\ 1 & 1-e^{-X_t\beta_2} & \beta_1 e^{-X_t\beta_2} X_t \\ 1 & 1-e^{-X_t\beta_2} & \beta_1 e^{-X_t\beta_2} X_t \\ 1 & 1-e^{-X_t\beta_2} & \beta_1 e^{-X_t\beta_2} X_t \\ 1 & 1-e^{-X_t\beta_2} & \beta_1 e^{-X_t\beta_2} X_t \end{bmatrix}$$

Motivation of Project

- Because **all of these approaches are approximations** of the nonlinear function, how well do they work in practice?

Method

<u>Simulation Design</u>

We utilized Paxton, et al. (2001) as a guide.

<u>The Generating Function (from du Toit & Cudeck 2001)</u>

$$Y_{it} = \beta_{0i} + (-1)(\beta_{1i} e^{-X_t \beta_{2i}} - 1) + \varepsilon_{it}$$

where: $\beta_0 = 20$ $(\phi_{11} = .25)$

$\beta_1 = 60$ $(\phi_{22} = .0625)$

$\beta_2 = .5$ $(\phi_{33} = .01)$ or $(\phi_{33} = 0)$ in the 'fixed-gamma' condition.

$\Theta_\varepsilon = .25$

<u>Conditions</u>

- Type of Modeling Approach (quadratic, cubic, fully latent, conditionally linear or linearized by Taylor expansion.).
- Whether β_2 was a fixed or random component.
- There were 1000 replications per condition

Results

Table 1: Schwartz's Bayesian Criterion and Percent Improper Solutions for Each Modeling Method

Model	Random Effects		Fixed β_2	
	BIC	Improper	BIC	Improper
Quadratic	8251.50	100%	7974.06	100%
Cubic	838.06	0%	577.17	100%
Fully Latent	6854.03	0%	-100.33	0.1%
Conditionally Linear	6839.29	0%	-118.02	0.1%
Linearization	1525.37	0%	881.49	100%

Conclusions

1. Nonlinearity of functional form and particularly in the parameters is an issue that researchers in the social and behavioral sciences must address.
2. LCM cannot estimate complex nonlinear functions directly, but there are four methods for modeling nonlinearity (polynomial, fully latent, conditionally linear and linearized models).
3. Comparing the five different estimating models that were used we found:
    - The quadratic is always inappropriate.
    - Surprisingly, the Cubic model fits the best in the fully random condition.
    - Only the conditionally linear and the fully latent gave proper solutions in the fixed β_2 condition.
4. Ultimately, because there is no nesting among the models, the choice of model is a philosophical decision that places the responsibility upon the researcher to find **congruence between the theoretical model of interest and the statistical model used to analyze the data**. Therefore, the interpretation of the parameters of the models is of the utmost interest to the applied researcher.

Interpretation of Parameters

Quadratic & Cubic Models

- Loadings fixed, so there is no interpretation of Λ.
- β_0: initial value at time = 0.
- β_1: rate of change per unit time.
- β_2: rate of change in rate of change per unit time.
- β_3: rate of change in the rate of change of the rate of change per unit time.

Fully Latent Model

- Interpret the free loadings (λ_{22} - λ_{52}) as proportional change during the time period.
- β_0: initial value at time = 0.
- β_1: overall change during time period.

Conditionally Linear

- β_0: initial value at time = 0.
- β_1: potential change during time period.
- β_2: exponential growth rate.

Linearization Model

- β_0: initial value at time = 0.
- β_1: potential change during the time period.
- β_2: exponential rate of growth.