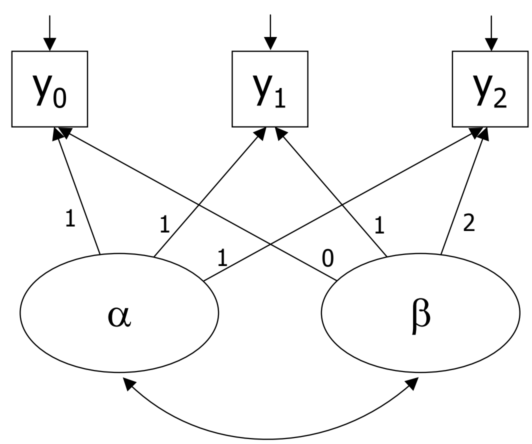
On the Use of Confidence Bands in Latent Trajectory Models

Daniel J. Bauer Patrick J. Curran
&
Kenneth A. Bollen

June 22, 2001

University of North Carolina at Chapel Hill

The Unconditional Latent Trajectory Model



Multilevel Representation

Level 1: $y_{it} = \alpha_i + \beta_i \lambda_t + \varepsilon_{it}$

Level 2: $\alpha_i = \mu_\alpha + \zeta_{\alpha_i}$

$\beta_i = \mu_\beta + \zeta_{\beta_i}$

Reduced Form Equation

$y_{it} = [\mu_\alpha + \mu_\beta \lambda_t] + [\zeta_{\alpha_i} + \zeta_{\beta_i} \lambda_t + \varepsilon_{it}]$

Substantive Example

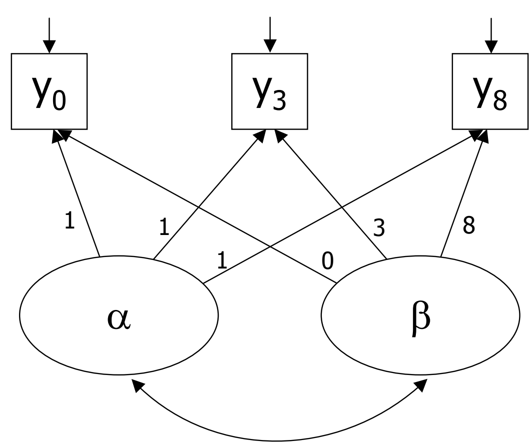
Sample

- Data from the *Dunedin Multidisciplinary Health & Development Study* courtesy of Terrie Moffitt and Avshalom Caspi.
- Birth cohort *N* = 1037 males and females assessed in 1972-1973.
- Current data: All males at age 18, 21, & 26: *N* = 445.

Measures

- Conduct Problems: Sum of presence of 8 delinquent acts (e.g., carried or used a weapon, physical fights, fire setting).
- Age 18 DSM-IIIR Depression Diagnosis: 88% = 0, 12% = 1
- Age 18 Alcohol Use Symptoms: Sum of 17 3-level Likert scale measures of negative alcohol consequences (e.g., hurt while drinking, family and friends object to drinking).

Unconditional LTM of Conduct Problems



Model Fit

$\chi^2(1)=7.5$, p=.005

RMSEA = .12, CFI=.99

Parameter Estimates

- Significant mean intercept of 1.87
- Significant group mean slope of -.05
- Significant variance of intercept and slope
- Significant correlation between intercept and slope of -.47

Conclusions and Lingering Questions

Conclusions from LTM analysis

• The estimated level of conduct problems at age 18 is significantly greater than 0.

• Conduct problems are, on average, decreasing at a significant rate over time.

• There is significant individual variability in starting levels of conduct problems and their rate of change over time.

• Adolescents with high levels of conduct problems at age 18 generally show the steepest declines in conduct problems over time.

Questions that Remain

• How precise is my estimate of the average trajectory?

• Are estimated levels of conduct problems significantly greater than 0 at all ages between 18 and 26?

These questions can be answered from the same model estimates by placing confidence bands on the mean trajectory.

The Logic of Confidence Bands

• Asymptotically, model parameter estimates are normally distributed.

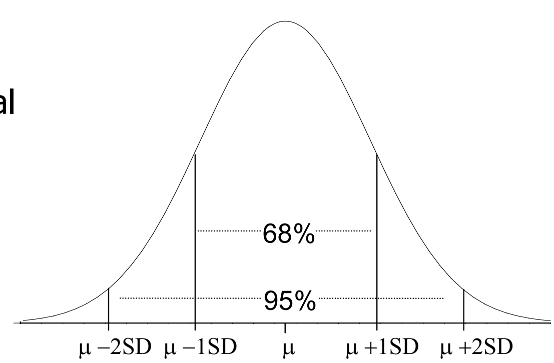
• Any linear combination y of normally distributed variables x

$$y = a_1 x_1 + \cdots + a_p x_p$$

is itself normally distributed with variance

$$VAR(y) = \sum_{i=1}^{p} \sum_{j=1}^{p} a_i a_j \sigma_{ij}$$

• 95% of the values of a normal distribution fall within 1.96 SDs of the mean



Using Confidence Bands in the Unconditional Model

Questions about the mean trajectory

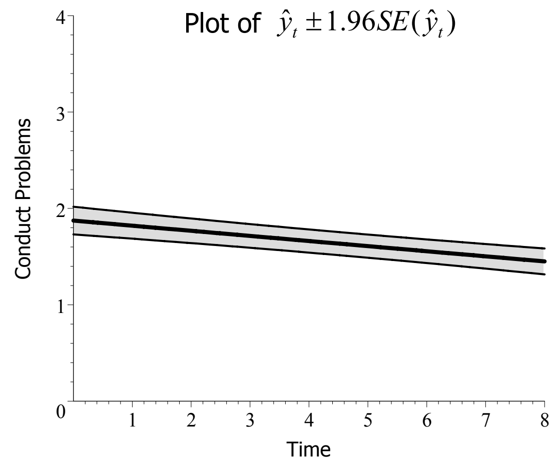
- How precise is my estimate of the mean trajectory of conduct problems?
- Do adolescents "age-out" of conduct problems so that the average level of conduct problems no longer significantly differs from 0?

Linear combination of interest

$$\hat{y}_t = \hat{\mu}_\alpha + \hat{\mu}_\beta \lambda_t$$

$$VAR(\hat{y}_t) = VAR(\hat{\mu}_\alpha) + 2\lambda_t COV(\hat{\mu}_\alpha, \hat{\mu}_\beta) + \lambda_t^2 VAR(\hat{\mu}_\beta)$$

Non-Simultaneous 95% CI for Mean Trajectory



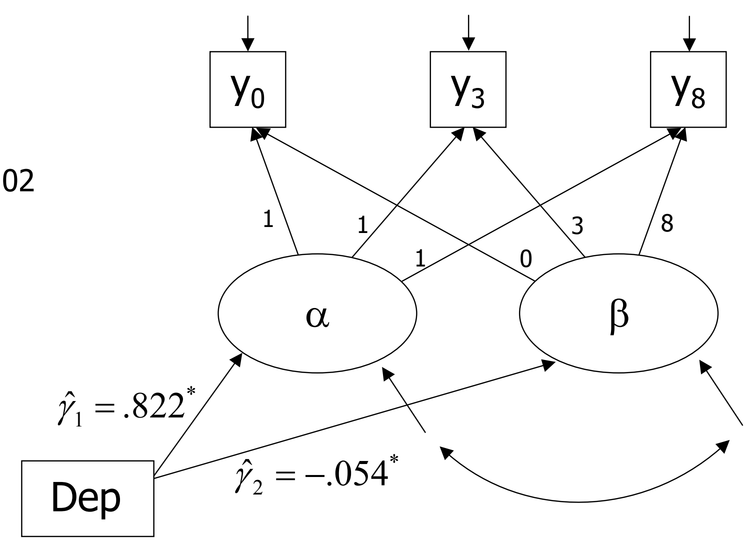
The Conditional LTM With A Dichotomous Predictor: Depression Diagnosis

Model Fit

$\chi^2(2)=8.4$, p=.02

RMSEA = .08

CFI=.99



Conclusion: Adolescents who are diagnosed as depressed at age 18 show higher levels of conduct problems initially, but their conduct problems also decline more rapidly over time.

Using Confidence Bands With Dichotomous Predictors

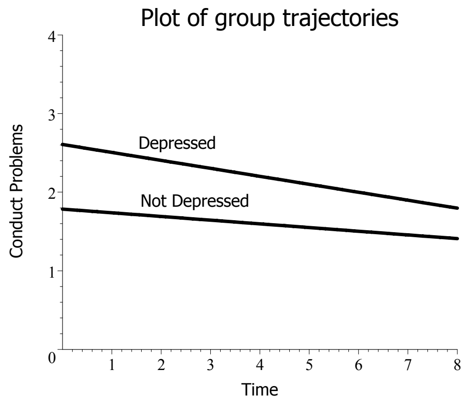
What else do we want to know?

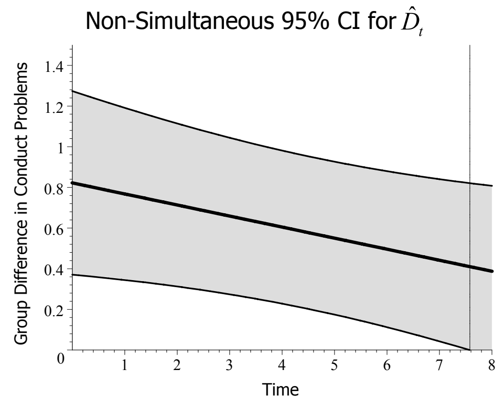
- At what ages are conduct problems significantly different for depressed ($dep = 1$) and non-depressed ($dep = 0$) adolescents?

Linear combination of interest

$$\hat{D}_t = \hat{y}_t\big|_{dep=1} - \hat{y}_t\big|_{dep=0} = \hat{\gamma}_1 + \hat{\gamma}_2\lambda_t$$

$$VAR(\hat{D}_t) = VAR(\hat{\gamma}_1) + 2\lambda_t COV(\hat{\gamma}_1, \hat{\gamma}_2) + \lambda_t^2 VAR(\hat{\gamma}_2)$$





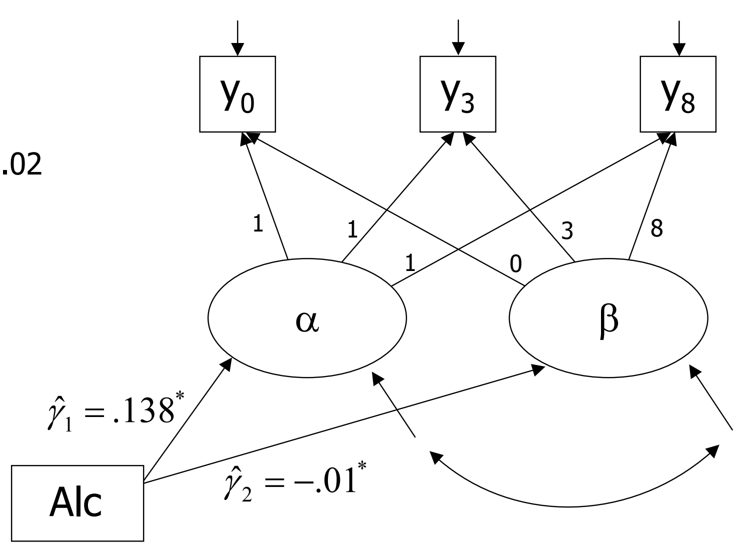
The Conditional LTM With A Continuous Predictor: Alcohol Use Symptoms

Model Fit

$\chi^2(2)$=8.3, p=.02

RMSEA=.08

CFI=.99



Conclusion: Adolescents with a high number of alcohol use symptoms at age 18 have higher levels of conduct problems initially, but their conduct problems also decline more rapidly over time.

Using Confidence Bands with Continuous Predictors

What else do we want to know?

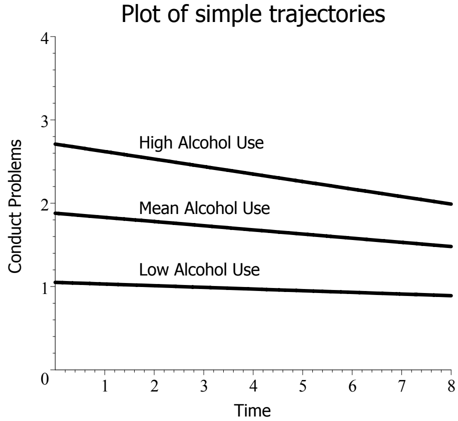
- Are there levels of alcohol use (x) at which declines in conduct problems ($\hat{\beta}$) no longer take place at a significant rate?

Linear combination of interest

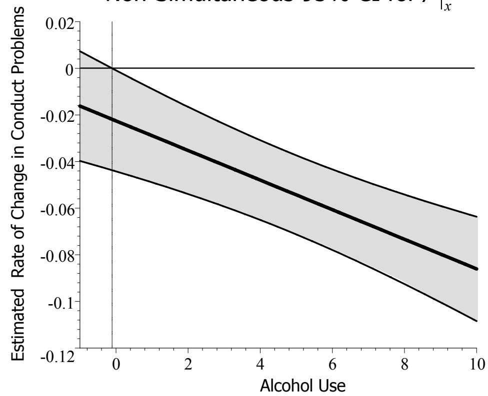
$$\hat{\beta}\Big|_x = \hat{\mu}_\beta + \hat{\gamma}_2 x$$

$$VAR(\hat{\beta}\Big|_x) = VAR(\hat{\mu}_\beta) + 2xCOV(\hat{\mu}_\beta, \hat{\gamma}_2) + x^2 VAR(\hat{\gamma}_2)$$

Plot of simple trajectories

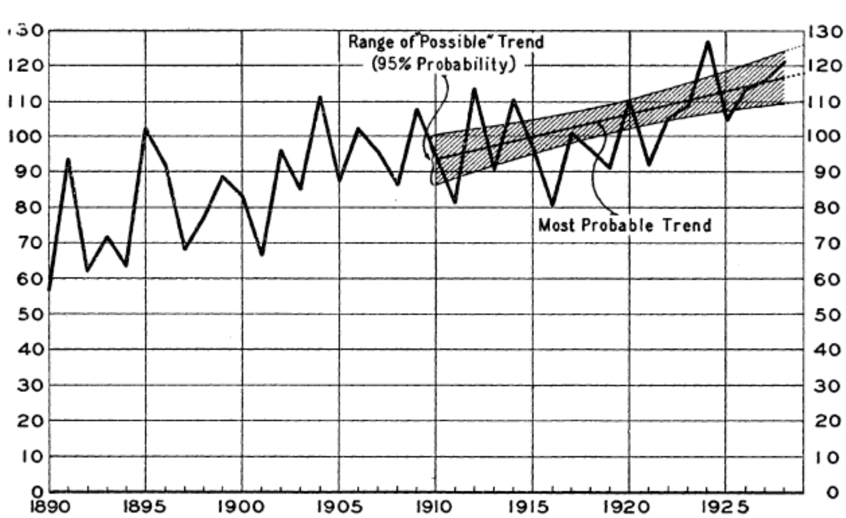


Non-Simultaneous 95% CI for $\hat{\beta}\Big|_x$



Not Completely a Novel Concept…

Working H. & Hotelling H. (1929). Applications of the theory of error to the interpretation of trends. *Journal of the American Statistical Association Supplement (Proceedings), 24*, 73-85.



ANNUAL AVERAGE YIELD OF POTATOES IN THE UNITED STATES AND RECENT TREND
(Bushels per Acre)

Why Use Confidence Bands with Latent Trajectory Models?

- Convey more information than plots of model-implied mean trajectories alone.

- Adds to other methods for probing interactions between time and other predictors in conditional growth models.

- Can be used to identify regions of significance for time or other predictors.

- Increases the range of substantive questions that can be addressed with the latent trajectory model.

- Can be extended to any polynomial growth model.

- Can be extended to multilevel growth modeling.