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Building Path Diagrams for Multilevel Models

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Multilevel models have come to play an increasingly important role in many areas of social science research. However, in contrast to other modeling strategies, there is currently no widely used approach for graphically diagramming multilevel models. Ideally, such diagrams would serve two functions: to provide a formal structure for deriving the underlying equations and to provide a mechanism for clearly and efficiently communicating the model structure, assumptions, and empirical results. Here the authors propose a path diagramming approach for multilevel models that seeks to meet these goals. The authors begin with a description of the core components of their proposed diagramming system and establish tracing rules for the direct derivation of model equations. They then demonstrate their approach using several published empirical multilevel applications and conclude with potential limitations and directions for future developments.

Keywords: multilevel modeling, hierarchical linear modeling, path diagrams, graphics

Multilevel models are being used at an increasing rate by many social scientists for the advantages they afford in the analysis of clustered or nested data. These models augment the more familiar general linear model through the incorporation of random effects at each of multiple levels of sampling. For instance, for hierarchically clustered data, one random effect might capture variability within groups, whereas a second might capture variability between groups. More complex multilevel models allow variability between groups in the effects of one or more predictors. These models can be applied to cross-sectional or longitudinal data structures and now represent a significant analytic tool available to the social science researcher.

In a typical empirical report, the specific structure of the multilevel model is conveyed descriptively within the text or (less frequently) through the presentation of formal model equations. The sample results are commonly presented in tabular form and often only for a subset of empirical findings. Indeed, we have relied on this strategy ourselves to provide a compact summary of model results (e.g., Bauer & Curran, 2005; Curran, Bauer, & Willoughby, 2006; Curran, Edwards, Wirth, Hussong, & Chassin, 2007). However, on the basis of our own experience as contributors, reviewers, and consumers of substantive multilevel applications, we believe that authors too often forgo complex equations in favor of tables that neither fully convey the specific structure of the estimated model nor provide a complete reporting of all relevant empirical results. This in turn occludes an understanding of both the model that was estimated and the substantive implications of the corresponding results.

As an alternative rhetorical device for conveying model results, graphs and diagrams are often preferable to tables and textual descriptions (e.g., Wainer & Thissen, 1981). We believe that this is also true for communicating the structure and underlying assumptions of the statistical model itself. Ideally, given a set of diagramming rules, the model equations should imply a single diagram, and conversely, the diagram should represent a unique set of corresponding mathematical equations (e.g., Boker, McArdle, & Neale, 2002). The equations and diagram are then alternative yet equivalent representations of the model.

An early sophisticated diagramming approach meeting these criteria was developed by Wright (1934) for path analysis. This was a truly remarkable approach in that the model-implied moment structure of the observed variables could be unambiguously derived directly from a graphical representation of the model. Over time, these path diagrams were expanded for use with a broad class of factor analysis and structural equation models (SEMs) with latent variables. Although occasionally criticized (e.g., Freedman,

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1987; Rogosa, 1987), the development and refinement of these path diagrams has clearly fostered the development, application, and dissemination of these models in social science research.

In our view, there are three primary reasons why researchers have adopted path diagrams for SEM applications. First, a path diagram enhances the communication of both the structure and sample results for complex models to a broad audience of readers. Second, a path diagram helps map a statistical model onto a theoretical substantive model; in this way a path diagram can aid in the specification of a statistical model that optimally corresponds to the theoretical model of interest (e.g., Bauer, 2003; Curran & Willoughby, 2003; Wohlwill, 1991). Finally, a path diagram helps to make model assumptions more explicit; for example, whereas correlated residuals are clearly presented in a path diagram, these are often less obvious (if not entirely absent) in a set of equations. Despite these advantages, no single diagramming strategy has achieved wide use for depicting multilevel models.

Presentations of path diagrams for multilevel models have appeared previously in several publications, and most involve adaptations of SEM path diagrams. One approach, used similarly by Bauer (2003), Curran (2003), and Mehta and Neale (2005), is to recast the multilevel model as an SEM so that an SEM path diagram can be used to represent the model. This approach has the distinct advantage that SEM diagramming rules continue to apply, but it is our opinion that the resulting diagrams are less intuitive than they might otherwise be. For instance, random effects are represented as latent variables for which the factor loadings are defined by observed "definition" variables, yet other observed variables are represented by boxes, and other latent variables are not actually considered to be random effects. Earlier, McArdle and Hamagami (1996) proposed using multiple groups SEM path diagrams to represent multilevel models. Although this approach also allows for the application of standard SEM diagramming rules, it requires the use of phantom latent variables that may be intimidating and nonintuitive for some users.

Also relevant are diagramming systems developed for multilevel SEMs by McDonald (1994), L. K. Muthén and Muthén (2003), B. O. Muthén and Satorra (1989), and Skrondal and Rabe-Hesketh (2005). Although clearly useful for their intended purposes, none of these diagramming approaches has been widely adopted for the most typical type of multilevel model: a model with fixed predictors, random coefficients, and no latent variables. There are two possible reasons for the lack of broad adoption of these diagrams. First, it is not always obvious how to simplify the diagrams to provide intuitive depictions for more standard multilevel models that do not contain latent variables. Second, these diagrams are designed primarily to communicate model structure and are not intended to provide a direct visual translation of the model equations or a modality for the presentation of results. In juxtaposition to these SEM-based approaches, we are aware of only one path diagram uniquely designed to represent a multilevel model (Kreft & de Leeuw, 1998, p. 72), but this was applied to a single specific case and has not been used elsewhere.

Our goal is thus to propose a comprehensive rule-based approach to path diagramming multilevel models that meets three primary aims. First, the diagram should be designed in a way that makes the graphical specification of the multilevel model as intuitive as possible. Second, a unique set of model equations should be directly derivable from the diagram. Finally, the diagram should provide a mechanism with which to clearly and unambiguously communicate even complex empirical results to a broad audience of researchers. To meet these aims, we first present a general framework for constructing a multilevel path diagram. We then build diagrams for various multilevel models of increasing complexity. Next we describe an algorithm for deriving both the multilevel and reduced-form equations for a given model. Finally, we apply our diagrams to three published multilevel applications and conclude with potential limitations and future directions.

To be clear at the outset, we are solely focused on what is often considered the "standard" multilevel linear model or the random coefficients/fixed regressors model. Importantly, we do not address recent developments in multilevel factor analysis or structural equation modeling (e.g., Bentler & Liang, 2003; Du Toit & Du Toit, 2005; Goldstein & McDonald, 1988; McDonald, 1994; Mehta & Neale, 2005; Rabe-Hesketh, Skrondal, & Pickles, 2004). Our restricted focus is quite intentional, as we desire to develop a system that is uniquely suited for the types of models most widely used in practice. Although we offer brief introductions to various multilevel models, we presume throughout that the reader already has a basic knowledge of the core components of the multilevel linear model (see, e.g., Bock, 1989; Goldstein, 2003; Hox, 2002; Kreft & de Leeuw, 1998; Longford, 1993; Raudenbush & Bryk, 2002; Snijders & Bosker, 1999).

Constructing a General Framework for a Multilevel Path Diagram

The diagramming method we propose here borrows from several existing systems but does not require knowledge of these other systems to be applied by the user. The current method also incorporates a number of unique components designed to capture effects not present in other types of modeling strategies.

Boxes

As is standard in path analysis and SEM diagrams, a box represents a measured variable that can serve as either a predictor or a criterion. Within this box, either an alphanumeric symbol or text label is used to name the observed variable. The variable name is presented in plain font if the variable is uncentered, it is presented in bold font if the variable is group-mean centered, and it is presented in bold italic font if the variable is grand-mean centered. Because the issue of centering rarely, if ever, applies to the dependent variable, this is always presented in plain font to denote that the dependent variable is uncentered.

Triangles

Borrowing from McArdle and Boker's (1991) RAM notation, a triangle labeled with the number "1" is used to define the intercept term in the equation; the label of "1" is used to reflect the column of 1's in the design matrix of the model that produces the intercept estimate (e.g., Raudenbush & Bryk, 2002, Equation 3.22). Because the multilevel model incorporates intercept terms at multiple levels of analysis, typically more than one triangle is used within a diagram. As such, the number "1" is subscripted to denote specific level (e.g., 1_1 and 1_2 indicating intercepts at Level 1 and Level 2, respectively).

Circles

A circle is used to represent a random coefficient and is labeled within the diagram accordingly. Our use of circles in the proposed diagramming strategy takes on a different meaning from those used in SEM diagramming schemes: Whereas a circle is used in SEM diagrams to represent an unobserved latent factor, here the circle represents an unobserved random effect. As in many SEM path diagrams, we do not use a circle to denote a residual term associated with an observed variable.

Straight Single-Headed Arrows

As in path analysis and SEM diagrams, a straight single-headed arrow represents a regression parameter. This effect is assumed to be fixed unless superimposed with a circle, in which case it is random. Single-headed arrows can represent four general types of relations: (a) the arrow can begin with a triangle and end with a circle or a square denoting an intercept, (b) the arrow can begin with a square and end with a square denoting the regression of one measured variable on another, (c) the arrow can begin with a square and end with a circle denoting the regression of a lower level random coefficient on a higher level measured variable, or (d) the arrow can begin with nothing and end with either a square or a circle denoting a residual or disturbance (as in SEMs, the implicit value of this path is 1).

Multiheaded Arrows

Borrowing from general SEM path diagrams, a multiheaded arrow indicates a covariance. These typically appear as curved double-headed arrows connecting two random effects. Importantly, only covariances which are estimated as model parameters are shown; as such, no covariances among exogenous predictors are displayed as these values are not estimated in fitting the model.

Applying the Diagramming Scheme to Various Multilevel Models

With the components described above, any given multilevel model can be translated directly into a diagram that uniquely represents the model structure. We now demonstrate this by diagramming a variety of multilevel models of varying complexity. In each case, we annotate the diagrams with the notation from the equations. Once greater familiarity has been established with the diagramming system, such annotation becomes unnecessary.

Two-Predictor Regression With Fixed Intercepts and Fixed Slopes

As a starting point, we apply the basic elements of the diagramming strategy to a standard two-predictor fixed effects regression. The single-level equation is given as

$$y_i = \beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + r_i, \tag{1}$$

where y is the outcome measured on individual i; β_0, β_1, and β_2 are fixed regression parameters relating the two predictor variables x_{1i} and x_{2i} to the response variable; and r_i is a random residual term with a mean of zero and variance σ^2 (i.e., $r_i \sim N[0, \sigma^2]$). The diagram for this model is presented in Figure 1. Several of the key components of the diagramming strategy are immediately apparent. First, the two predictors and one criterion are expressed in boxes indicating that these are measured variables. Second, the labels for the two predictors are in plain font indicating that these are in the original metric of the measure (i.e., are uncentered). Third, the single-headed arrow from each predictor to the criterion reflects the regression of the criterion on each predictor. Finally, the single-headed arrow from the triangle

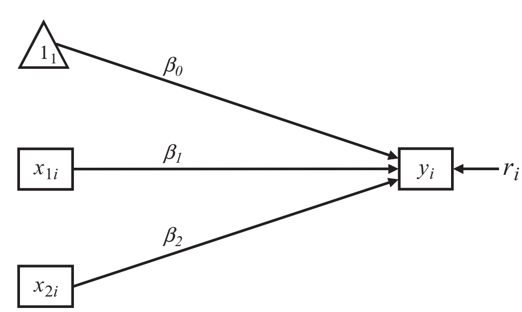

Figure 1. Two-predictor fixed-effects regression with one intercept and two slopes.

to the criterion indicates that there is the usual regression intercept estimated for the dependent measure.

Also notable is what is absent from the diagram. In the standard SEM path diagram there would be a curved arrow between the two predictors to reflect that these measures are correlated. However, for the reasons described above, we omit these from the diagram. Further, there are no circles used in this diagram. This explicitly indicates that the residual term r is the only source of random variability in this model, and the intercept and slopes do not vary randomly over any higher grouping. Finally, there is no curved arrow between the residual and either predictor, which clearly indicates that the residual errors are assumed to be uncorrelated with the predictors.

Two-Predictor Regression With Random Intercepts and Fixed Slopes

The most basic expansion of a fixed-effects regression model to a multilevel model is to allow the intercept term to vary randomly over groups. This parameterization implies that the regression slopes remain fixed (i.e., are invariant over groups), but the intercept term does not. The Level-1 model is given as

$$y_{ij} = \beta_{0j} + \beta_1 x_{1ij} + \beta_2 x_{2ij} + r_{ij}, \qquad (2)$$

where i represents individual and j represents group, and the Level-2 model is

$$\beta_{0j} = \gamma_{00} + u_{0j}. \qquad (3)$$

The random intercept (denoted β_{0j}) is thus expressed as an additive function of a grand mean (γ_{00}) and a group-level deviation from this mean (u_{0j}). The within-group residual variance is denoted σ^2 (i.e., $r_{ij} \sim N[0, \sigma^2]$), and the between-group variance is denoted τ_{00} (i.e., $u_{0j} \sim N[0, \tau_{00}]$). The Level-1 and Level-2 distinction is primarily pedagogical, and from a statistical standpoint we are interested in the reduced-form expression. The reduced form is derived by the simple substitution of Equation 3 into Equation 2, which results in

$$y_{ij} = (\gamma_{00} + \beta_1 x_{1ij} + \beta_2 x_{2ij}) + (u_{0j} + r_{ij}). \qquad (4)$$

The path diagram for this model is presented in Figure 2. There are three key differences between Figure 1 and Figure 2. First, the two Level-1 predictors are now presented in italicized font indicating these are group-mean centered (in contrast to being uncentered in Figure 1). Second, a circle labeled β_{0j} has been superimposed on the path linking the triangle to the dependent variable y_{ij}. This addition indicates that the intercept term varies randomly over groups, whereas the other two regression parameters remain fixed (as indicated by the lack of an imposed circle on either path). Third, an arrow points to the circle from a second

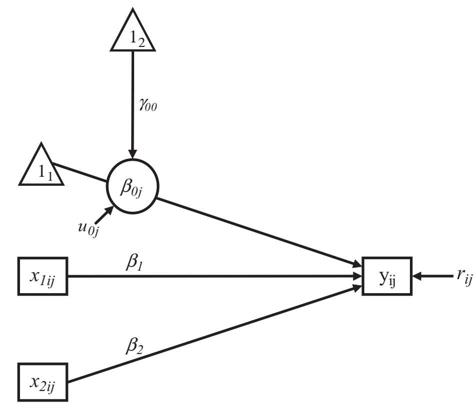

Figure 2. Two-predictor regression with a random intercept and two fixed slopes.

triangle implying an intercept (γ_{00}) for the random intercept term (β_{0j}). The group-level deviations from this intercept are represented by the arrow from the disturbance u_{0j}. The subscripting of 1_1 and 1_2 within each triangle clearly differentiates the Level-1 and Level-2 intercept terms, respectively. Finally, the absence of a curved arrow between the Level-1 and Level-2 disturbances explicates the assumption that these are uncorrelated.

Two-Predictor Regression With Random Intercept and Random Slopes

We next extend the random intercept model to include random slopes for the two predictors. The Level-1 model is now

$$y_{ij} = \beta_{0j} + \beta_{1j} x_{1ij} + \beta_{2j} x_{2ij} + r_{ij}; \qquad (5)$$

the Level-2 model is

$$\beta_{0j} = \gamma_{00} + u_{0j}$$
$$\beta_{1j} = \gamma_{10} + u_{1j}$$
$$\beta_{2j} = \gamma_{20} + u_{2j}; \qquad (6)$$

and the reduced-form expression is

$$y_{ij} = (\gamma_{00} + \gamma_{10} x_{1ij} + \gamma_{20} x_{2ij}) + (u_{0j} + u_{1j} x_{1ij} + u_{2j} x_{2ij} + r_{ij}). \qquad (7)$$

The random intercept (β_{0j}) and the two random slopes (β_{1j} and β_{2j}) are expressed as additive functions of their grand means (the γ's) and group-level deviations from these means (the u's). The multivariate distribution of the vector of group-level deviations is assumed multivariate normal (e.g., $\mathbf{u}_j \sim N[0, \mathbf{T}]$), where $\mathbf{T}$ is the covariance matrix of the Level-2 deviations.

The corresponding path diagram presented in Figure 3

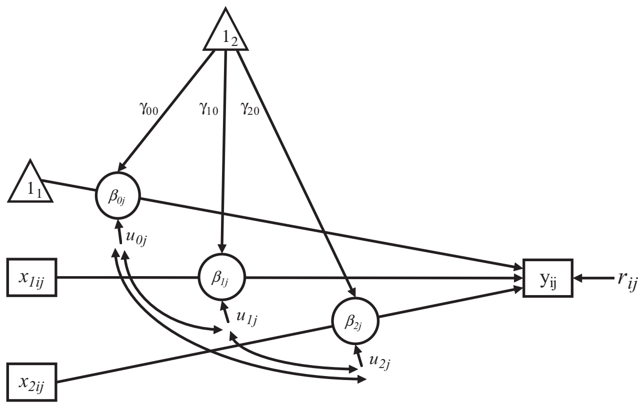

Figure 3. Two-level random-effects regression model with three correlated random effects at Level 2.

now includes a circle superimposed not only on the intercept term but also on each of the regression parameters. As with the prior figure, the two Level-1 predictors are presented in italicized font to denote group-mean centering. All three random effects are regressed upon the triangle to denote the estimation of the intercepts for each random effect (i.e., γ_{00}, γ_{10}, and γ_{20} from Equation 6). Further, the group-level deviations of the random effects from the grand-mean effects are represented by the arrows marked with the corresponding u's. A new feature of this diagram is that the disturbances for all three random effects are connected by curved two-headed arrows indicating that the covariance matrix of the random effects is unrestricted in this model (i.e., all elements of **T** are estimated). Fewer curved arrows would reflect a more restricted structure for **T** (e.g., no curved arrows at all would reflect that **T** is diagonal).

Multilevel Model With Both Level-1 and Level-2 Predictors

Multilevel models often include predictors measured at the group level. For example, consider an extension of the previous model in which two Level-2 predictors are included as predictors of each random effect. The Level-1 equation is again

$$y_{ij} = \beta_{0j} + \beta_{1j}x_{1ij} + \beta_{2j}x_{2ij} + r_{ij}, \tag{8}$$

but the Level-2 equations are now expanded to include the two group-level predictors w_{1j} and w_{2j}:

$$\begin{aligned}\beta_{0j} &= \gamma_{00} + \gamma_{01}w_{1j} + \gamma_{02}w_{2j} + u_{0j}\\ \beta_{1j} &= \gamma_{10} + \gamma_{11}w_{1j} + \gamma_{12}w_{2j} + u_{1j}\\ \beta_{2j} &= \gamma_{20} + \gamma_{21}w_{1j} + \gamma_{22}w_{2j} + u_{2j}.\end{aligned} \tag{9}$$

Substituting Equation 9 into Equation 8 and collecting terms results in the reduced-form expression given as

$$\begin{aligned}y_{ij} = {} & (\gamma_{00} + \gamma_{10}x_{1ij} + \gamma_{20}x_{2ij} + \gamma_{01}w_{1j} + \gamma_{02}w_{2j})\\ & + (\gamma_{11}w_{1j}x_{1ij} + \gamma_{12}w_{2j}x_{2ij} + \gamma_{21}w_{1j}x_{2ij} + \gamma_{22}w_{2j}x_{2ij})\\ & + (u_{0j} + u_{1j}x_{1ij} + u_{2j}x_{2ij} + r_{ij}).\end{aligned} \tag{10}$$

We present this same model in Figure 4, which highlights the capability of the diagram to convey the identical model structure independently of the increasingly complex model equations. Note that the diagram explicates the covariance structure among the random effects that is not reflected in Equations 8, 9, and 10. The path diagram also reflects that there is a total of nine fixed effects: three associated with the intercept, three with w_{1j}, and three with w_{2j}. Whereas the Level-1 predictors are in italicized font to reflect that these are group-mean centered, the Level-2 predictors are in bold and italicized font to reflect that these are grand-mean centered. Further, whereas the random effects regression model shown in Figure 3 included unconditional random effects at Level 2 (i.e., there were no Level-2 predictors), these are now conditional (or residual) random effects given the presence of the two Level-2 predictors. Figure 4 also highlights the cross-level interactions between the Level-1 and Level-2 predictors, which are not explicated in Equations 8 and 9 (but are evident in the reduced-form expression in Equation 10). Specifically, the two Level-2 measures predict the slopes of the Level-1 measures and thus exert a "cross-level" influence on the outcome.[1]

Three-Level Model With Multiple Predictors at all Levels

One particularly elegant aspect of the multilevel model is the ease with which these models can be expanded to incorporate higher orders of nesting. Although in principle there can be as many potential levels of nesting as an empirical data set can support, here we only consider three (see Raudenbush & Bryk, 2002; Snijders & Bosker, 1999, for further details on three-level models). We consider a specific three-level model in which there are two predictors at each of the three levels. Consistent with Raudenbush and Bryk (2002), we use new notation to define the Level-1 model and expand the prior notation for the Level-2 and Level-3 models.

The criterion is now denoted y_{ijk} to denote the measured

[1] In some circumstances, it is possible to estimate a cross-level interaction in the absence of a lower order random effect (e.g., Raudenbush & Bryk, 2002, p. 28, Equation 2.20). Such an effect can be clearly represented in the diagram by simply drawing a single-headed arrow from the Level-2 predictor to midspan of the single-headed arrow from the Level-1 predictor to the outcome. All tracing rules would apply as usual.

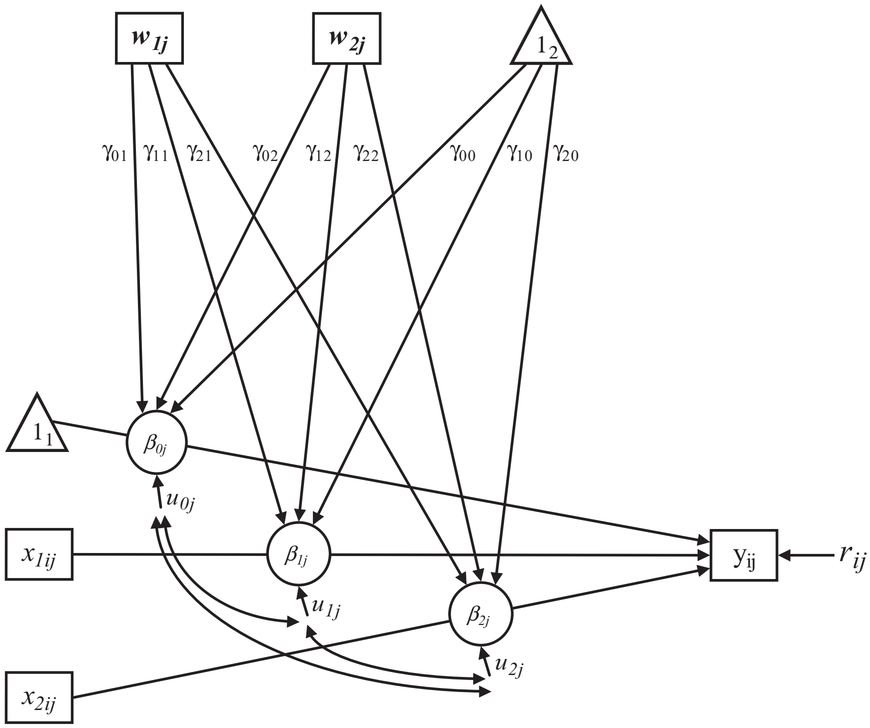

Figure 4. Two-level model with two Level-1 predictors, two Level-2 predictors, and three correlated random effects at Level 2.

value of y for observation i nested within group j which itself is nested in group k. The Level-1 equation for our example is given as

$$y_{ijk} = \pi_{0jk} + \pi_{1jk}a_{1ijk} + \pi_{2jk}a_{2ijk} + e_{ijk}, \tag{11}$$

where a_{1ijk} and a_{2ijk} are the two Level-1 predictors; π_{0jk}, π_{1jk}, and π_{2jk} are the Level-1 intercept and slopes, respectively; and e_{ijk} is the residual. For example, this might represent a growth model in which time is nested within child who is in turn nested within family, and a_{1ijk} and a_{2ijk} represent the linear and quadratic components of the growth trajectory, respectively (e.g., time and time squared).

The Level-2 equation is then expressed in the notation familiar from the two-level model presented earlier such that

$$\pi_{0jk} = \beta_{00k} + \beta_{01k}x_{1jk} + \beta_{02k}x_{2jk} + r_{0jk}$$

$$\pi_{1jk} = \beta_{10k} + \beta_{11k}x_{1jk} + \beta_{12k}x_{2jk} + r_{1jk}$$

$$\pi_{2jk} = \beta_{20k}, \tag{12}$$

where x_{1jk} and x_{2jk} denote the Level-2 predictors unique to unit j within group k. Continuing our example, this might reflect that the intercept and linear components of the growth model vary randomly across multiple children nested within family, and these are each regressed on the two child-level predictors (e.g., child gender and child psychopathology). Finally, the Level-3 equations are defined as

$$\beta_{00k} = \gamma_{000} + \gamma_{001}w_{1k} + \gamma_{002}w_{2k} + u_{00k}$$

$$\beta_{01k} = \gamma_{010}$$

$$\beta_{02k} = \gamma_{020}$$

$$\beta_{10k} = \gamma_{100} + \gamma_{101}w_{1k} + \gamma_{102}w_{2k} + u_{10k}$$

$$\beta_{11k} = \gamma_{110}$$

$$\beta_{12k} = \gamma_{120}$$

$$\beta_{20k} = \gamma_{200}, \tag{13}$$

where w_{1k} and w_{2k} denote the Level-3 predictors unique to group k. Completing our example, this might reflect that the intercept and linear components of the growth trajectory are also predicted by the two family-level predictors (e.g., parent income and parent psychopathology). There may also be some similarity between the trajectories of children residing within the same family that is unexplained by the set of predictors, necessitating the inclusion of the family-level disturbance terms.

We can again derive the reduced-form expression by substituting Equation 13 into Equation 12, and Equation 12 into Equation 11 to result in the rather unwieldy expression

$$y_{ijk} = (\gamma_{000} + \gamma_{100}a_{1ijk} + \gamma_{200}a_{2ijk} + \gamma_{010}x_{1jk} + \gamma_{020}x_{2jk} + \gamma_{001}w_{1k} + \gamma_{002}w_{2k}) + (\gamma_{110}a_{1ijk}x_{1jk} + \gamma_{120}a_{1ijk}x_{2jk} + \gamma_{101}a_{1ijk}w_{1jk} + \gamma_{102}a_{1ijk}w_{2k}) + (u_{00k} + u_{10k}a_{1ijk} + r_{0jk} + r_{1jk}a_{1ijk} + e_{ijk}). \quad (14)$$

It is little wonder why many authors forgo including either the multilevel or reduced-form equations in their manuscripts when fitting such complex models.

Despite the complexity of this model, however, the very same relations can be fully represented in path diagram form, as seen in Figure 5. The diagram explicates the predictors at each level of the model. Further, the Level-1 measures a_{1ijk} and a_{2ijk} are labeled in plain font to reflect that these are scaled in the raw metric of time; the Level-2 measures x_{1jk} and x_{2jk} are labeled in italicized font to reflect these are group-mean centered within the Level-2 groups; and the Level-3 measures w_{1k} and w_{2k} are labeled in bold and italic font to reflect that these are grand-mean centered. The diagram also shows that, at Level 1, the intercept and the slope for a_{1ijk} vary across both Level-2 and Level-3 units. The effects of the Level-2 predictors are, however, clearly defined as fixed effects given the omission of circles superimposed on these pathways. The assumption that the random effects covary within but not across levels is also explicit given the placement of the curved double-headed arrows. Additionally, following the downward flow of the arrows, we can see that there will be main effects for all six predictors, as well as two-way cross-level interactions between the two Level-1 and two Level-2 predictors and between the two Level-1 and two Level-3 predictors, yielding a total of 11 fixed effects (this is explicated further in the tracing rules presented below). In total, Figure 5 equivalently represents the model given in Equation 14, yet we believe the path diagram offers a unique way to foster the understanding of the model at hand.

Summary

We have focused on five general structures of the multilevel model that are commonly encountered in practice. The proposed diagramming strategy can be applied to any combination of fixed or random effects models. Extensions to the standard two- and three-level models are equally amenable to the diagramming system. For instance, multiple outcome variables can be included to build diagrams for a variety of types of multivariate models. Similarly, a cross-

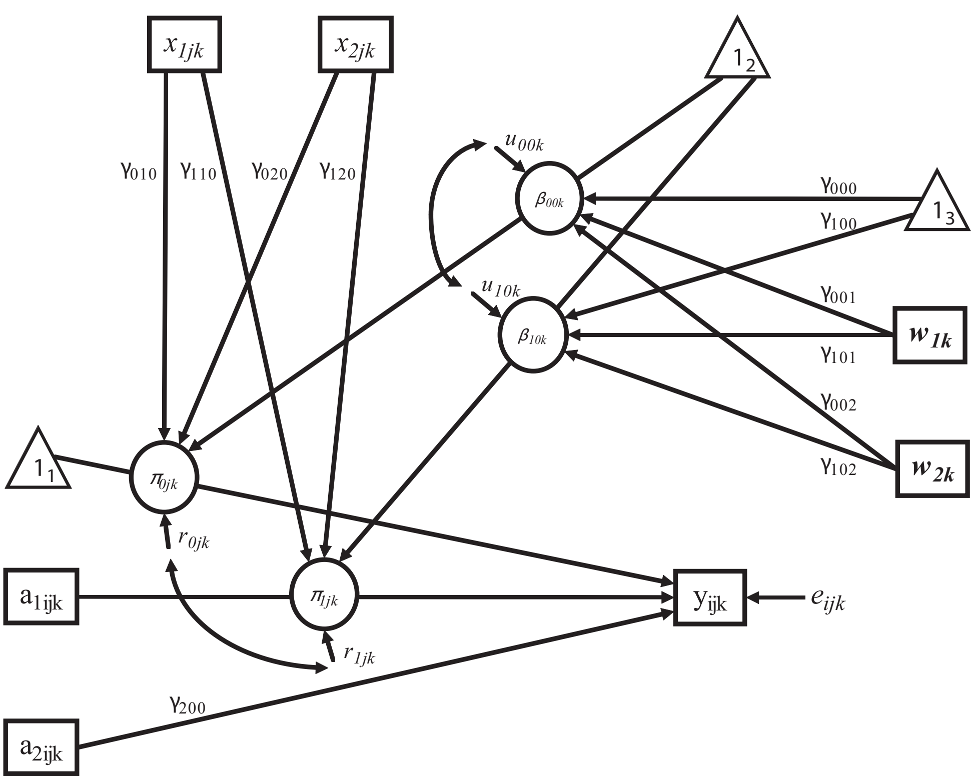

Figure 5. Three-level model with two predictors at each of three levels and two correlated random effects at Level 2 and Level 3.

classified model can be diagrammed by including different Level-2 disturbances for each dimension of the cross-classification over which a coefficient varies. Rather than catalog all of the possible model structures that can be diagrammed, we next explore in greater detail the two key purposes for which these diagrams can be used: to derive the equations that define the model and to unambiguously communicate empirical results to aid in substantive interpretation and broader dissemination of findings.

Using Path Diagrams for Deriving Model Equations

As we have demonstrated earlier, multilevel models can be equivalently expressed with either a set of multilevel equations (e.g., the Level-1 and Level-2 distinction) or a single reduced-form equation (e.g., in which the Level-2 equations are substituted into the Level-1 equation). There are distinct advantages to each form of expression, and the optimal selection depends upon the specific application at hand. In our experience, many researchers in the social sciences prefer to work with the multilevel expressions. This preference is likely due to a variety of reasons, not the least of which is the often intuitive expression of regression equations existing within each level of analysis. Despite this advantage, we believe that there are several important reasons why it is useful to more fully understand the reduced-form expression as well.

First, from a strictly pedagogical standpoint, it is important to appreciate that the dependent measure y_{ij} is simultaneously expressed as a function of all model parameters (e.g., Equation 10). This point is sometimes lost in the multilevel expressions (e.g., the influence of a Level-2 predictor does not explicitly appear in a Level-1 equation; e.g., Equations 8 and 9). Second, the reduced-form expression explicates the critical presence of cross-level interactions that are often not fully appreciated in the Level-1 and Level-2 equations (Bauer & Curran, 2005; Tate, 2004). Finally, from a strictly practical standpoint, there are a number of important software packages that require the multilevel model be expressed in terms of the reduced-form equation (e.g., SAS PROC MIXED, Version 9.1.3, or SPSS MIXED, Version 15.0). For these reasons, we believe it is important to develop a way to encourage the comfortable transition between the multilevel equations and the reduced-form equation and back again.

However, writing out the reduced-form equations from a multilevel expression can be a sometimes tedious and error prone task, especially for more complex models (e.g., consider the three-level model expressed in Equations 11, 12, 13, and 14 from above). This is particularly salient for individuals who may be less comfortable with manipulating mathematical equations yet desire to use the analytic techniques to evaluate important research hypotheses. As we demonstrate below, the reduced-form expression for even a complex multilevel model can be directly obtained from a path diagram by following a small number of basic rules. We first present tracing rules that allow for the derivation of the reduced-form expression from a given path diagram followed by slightly modified rules needed for deriving the multilevel equations from the same diagram. Although we focus on the two-level model, all of these rules directly generalize to the three-level model as well.

Tracing Rules for Deriving Reduced-Form Equations

There are four steps needed to derive the reduced-form equation from a given path diagram.

1. First, identify the intercept and all of the predictors from which only a single-headed arrow emanates and no circle is superimposed on the pathway. For example, in Figure 1 this would simply be the Level-1 triangle and two Level-1 boxes; in Figure 2, this would be the Level-2 triangle and the two Level-1 boxes; in Figure 3 this would only be the Level-2 triangle; and in Figure 4, this would be the Level-2 triangle and the two Level-2 predictors.

2. Next, trace from the intercept and each predictor identified in Step 1 through the diagram to end at the criterion measure y following two rules. First, multiply whatever is denoted by the triangle (e.g., 1) or the square (e.g., x_{1ij} or w_{1j}) by the corresponding path coefficient (e.g., γ_{00} or γ_{22}); then, if a circle is encountered in the trace, multiply the first product by whatever is denoted by the triangle or square at the *head* of the path with the circle. Steps 1 and 2 identify all of the fixed effects.

3. Then repeat the process described in Step 2, but starting from all residual terms (i.e., the u's and r_{ij}). This step identifies all of the random effects.

4. Finally, the reduced-form expression for the given model is simply the sum of all the traces derived from Steps 2 and 3.

As a demonstration, we apply these tracing rules to derive the reduced-form expression of the two-level model presented in Figure 4 and defined in Equation 10; each step is also presented in Figure 6. From the diagram we can immediately see that there will be a total of nine traces necessary to derive the fixed effects for the reduced-form equation (three traces associated with the Level-2 intercept, and three for each of the two predictors); further, there will be a total of four traces necessary to derive the random effects (three traces associated with the three random effects at Level 2, and one for the residual term).

First, we identify the intercept term and set of predictors from which a single-headed arrow emanates and upon which the path does not have a circle superimposed. In Figure 4, the Level-2 intercept and the two Level-2 predictors w_{1j} and w_{2j} fulfill these requirements. Next, we follow each possible trace through the diagram by multiplying the

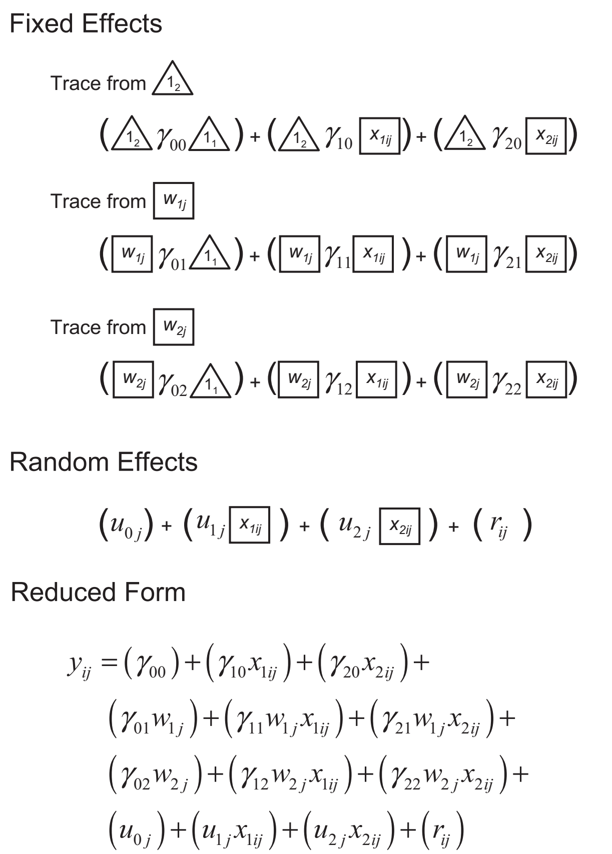

Figure 6. Step-by-step derivation of the reduced-form expression of a two-level model.

head of the trace with the corresponding coefficient and multiplying this product by the head of any lower level effect. So, starting with the Level-2 intercept, we compute three components of the reduced-form equation: $(1)(\gamma_{00})(1)$, $(1)(\gamma_{10})(x_{1ij})$, and $(1)(\gamma_{20})(x_{2ij})$. Importantly, note that when deriving the reduced-form equation, we are not interested in what is labeled within the circles (e.g., β_{0j}) but instead what lies at the *head* of the path on which the circle is superimposed. Because the value of 1 is implicit, the first contribution associated with the Level-2 intercept is simply

$$(\gamma_{00} + \gamma_{10}x_{1ij} + \gamma_{20}x_{2ij}). \quad (15)$$

We repeat this process for the next predictor w_{1j}. The three associated components are $(w_{1j})(\gamma_{01})(1)$, $(w_{1j})(\gamma_{11})(x_{1ij})$, and $(w_{1j})(\gamma_{21})(x_{2ij})$. These three contributions highlight the main effect contribution of w_{1j} along with the two cross-level interactions with x_{1ij} and x_{2ij}. Thus, the next three contributions are

$$(\gamma_{01}w_{1j} + \gamma_{11}w_{1j}x_{1ij} + \gamma_{21}w_{1j}x_{2ij}). \quad (16)$$

Finally, the three fixed effects associated with w_{2j} are traced just as they were for w_{1j} with the resulting contribution

$$(\gamma_{02}w_{2j} + \gamma_{12}w_{2j}x_{1ij} + \gamma_{22}w_{2j}x_{2ij}). \quad (17)$$

Equations 15, 16 and 17 reflect the fixed-effects part of the reduced-form expression of the model in Figure 4. We next add each residual term to compute the random-effects part of the model. In Figure 4 we start with each Level-2 deviation term (the *u*'s associated with each circle) and multiply each *u* by the value at the head of the corresponding pathway. Recall that the implicit value of these one-headed pathways is 1. The first term is thus $(u_{0j})(1)$ for the random intercept; the second term is $(u_{1j})(1)(x_{1ij})$ for the random slope associated with the first Level-1 predictor; and the third term is $(u_{2j})(1)(x_{2ij})$ for the random slope associated with the second Level-1 predictor. Finally, we add the Level-1 residual $(r_{ij})(1)$ to complete the random effects (the value of the path associated with r_{ij} is also implicitly 1). Taken together, the random-effects part of the reduced-form model is

$$(u_{0j} + u_{1j}x_{ij} + u_{2j}x_{2ij} + r_{ij}). \quad (18)$$

Collecting all of the above terms together, the reduced-form expression for the multilevel model expressed in Figure 4 is

$$y_{ij} = (\gamma_{00} + \gamma_{10}x_{1ij} + \gamma_{20}x_{2ij}) + (\gamma_{01}w_{1j} + \gamma_{11}w_{1j}x_{1ij} + \gamma_{21}w_{1j}x_{2ij}) + (\gamma_{02}w_{2j} + \gamma_{12}w_{2j}x_{1ij} + \gamma_{22}w_{2j}x_{2ij}) + (u_{0j} + u_{1j}x_{ij} + u_{2j}x_{2ij} + r_{ij}), \quad (19)$$

which exactly corresponds to the reduced-form equation presented in Equation 10. Each step is also summarized in Figure 6.

Tracing Rules for Deriving Multilevel Equations

The above steps allow for the derivation of the reduced-form equation. Modest modifications to the tracing rules described above are needed to allow for the derivation of the set of multilevel equations from a given path diagram. The key to deriving the multilevel equations is to view a multilevel model as having multiple dependent variables, the outcome variable *y* at Level 1 and the random coefficients at Level 2. Of course in actuality there is only a single outcome variable *y*, but a multilevel perspective allows for the clear disaggregation of Level-1 and Level-2 effects. Given this, the necessary modifications to the existing tracing rules are to (a) identify all squares and circles for which at least one single-headed arrow terminates and (b) identify all predictors that directly influence the specific square or circle.

We again consider the model presented in Figure 4. To derive the Level-1 equation, we need to identify the set of direct determinants of y_{ij}; that is, we must incorporate all traces that exist only within the Level 1 part of the model that lead directly to y_{ij}. Here we see there are four: one from the Level-1 intercept and one each from x_{1ij}, x_{2ij}, and r_{ij}. We start at the head of each of these paths, and we multiply what is denoted at the head with what is superimposed on the path as we proceed downstream to y_{ij}. These contributions are $(1)(\beta_{0j})$, $(x_{1ij})(\beta_{1j})$, $(x_{2ij})(\beta_{2j})$, and $(r_{ij})(1)$. These four components are then summed to represent the Level-1 equation

$$y_{ij} = \beta_{0j} + \beta_{1j}x_{1ij} + \beta_{2j}x_{2ij} + r_{ij}. \tag{20}$$

We then repeat this process for each component that can be considered another "dependent variable" in the model. In Figure 4, these are β_{0j}, β_{1j}, and β_{2j} and are identified as such given that they are influenced by at least one single-headed arrow. Each of these outcomes is defined to be influenced by four single-headed arrows: one from the Level-2 intercept, two from the two Level-2 predictors, and one from the Level-2 disturbance. A separate equation must be derived for each of these three outcomes. For example, the determinants of β_{1j} are $(1)(\gamma_{10})$, $(w_{1j})(\gamma_{11})$, $(w_{2j})(\gamma_{12})$, and $(u_{1j})(1)$. These are summed to result in

$$\beta_{1j} = \gamma_{10} + \gamma_{11}w_{1j} + \gamma_{12}w_{2j} + u_{1j}, \tag{21}$$

which is the Level-2 expression for the random slope associated with x_{1ij}. The same process would result in the relevant equations for β_{0j} and β_{2j}, respectively.

Using Path Diagrams to Aid in the Presentation of Model Results

We have demonstrated our first goal of developing a path diagram that can be used to easily derive either the multilevel or reduced-form equations for a given multilevel model. Our second goal is to use this same diagramming system to enhance the communication of complex empirical results to a broad audience of researchers. This may be particularly salient when considering consumers of research who are less familiar with the intricacies of multilevel modeling but nonetheless strive to gain a clear understanding of the model results from a substantive perspective.

To facilitate the clear presentation of a set of empirical results, we invoke two slight modifications to the diagrams to further explicate the relevant parameter estimates of interest. First, up to now we have denoted the Level-1 and Level-2 residuals as individual and group-specific deviations (e.g., r_{ij} and u_{0j}). However, the corresponding model parameters are the *variances* of these deviations (e.g., $var[r_{ij}] = \sigma^2$ and $var[u_{0j}] = \tau_{00}$). Thus, when reporting the random components from a given model application, we denote the numerical values of these variance estimates within the corresponding circle in place of the Greek notation used to define the given term (e.g., we replace "β_{0j}" with the sample estimate for the variance of the intercept). Second, if the researcher desires to indicate the statistical significance of the obtained estimates, we suggest that all sample estimates be presented but nonsignificant values be enclosed in parentheses (but see Harlow, Mulaik, & Steiger, 1997, for limitations of this method of presentation). Confidence intervals can also be presented in place of the sample point estimates. With these two modifications, we now turn to using these diagrams to summarize several previously published empirical applications.

Example 1: Sliwinski, Hofer, Hall, Buschke, and Lipton (2003)

Our first example focuses on a two-level growth model of memory decline in older adults presented in Sliwinski et al. (2003, Table 6, Model 3a, p. 665). The sample used for this model consisted of $N = 293$ elderly adults drawn from the longitudinal Bronx Aging Study. The subjects were free of dementia and were assessed annually over a period of 1–16 years (with an average follow-up of 5 years). The primary outcome of interest was a continuous measure of memory. The four Level-1 predictors were the linear and quadratic effects of chronological age and the linear and quadratic effects of time to attrition from the study, respectively. All Level-1 predictors are labeled in plain font to reflect that these are scaled in the raw metric of *age* and *time*. Random effects were estimated for the Level-1 intercept and all four Level-1 predictors. Because there were no Level-2 predictors, this can be considered a random effects regression model. This model is presented in Figure 7.

Several model features are immediately apparent in the diagram. First, there are five fixed effects, four of which are significantly different from zero ($p < .05$). Second, there is a Level-1 residual and five Level-2 random effects; the Level-1 residual is uncorrelated with the Level-2 random effects, and the Level-2 random effects all covary with one another.[2] Third, because the original article did not report the sample estimates for the variances or covariances of the random effects, these are denoted in the figure by a question mark. If available, these sample estimates would have been reported within the confines of the circles. This first example demonstrates the core components of the proposed diagramming system. We now extend these to two more complicated models.

Example 2: Jenkins, Rasbash, and O'Connor (2003)

We next consider a two-level model presented by Jenkins et al. (2003, Table 1, Model 7, p. 104). The data set was comprised of 8,096 children nested within 3,860 families; family size ranged between two and four children. The

[2] Although the specific covariance structure was not explicated in Sliwinski et al. (2003), we are inferring this structure from other models presented in the manuscript.

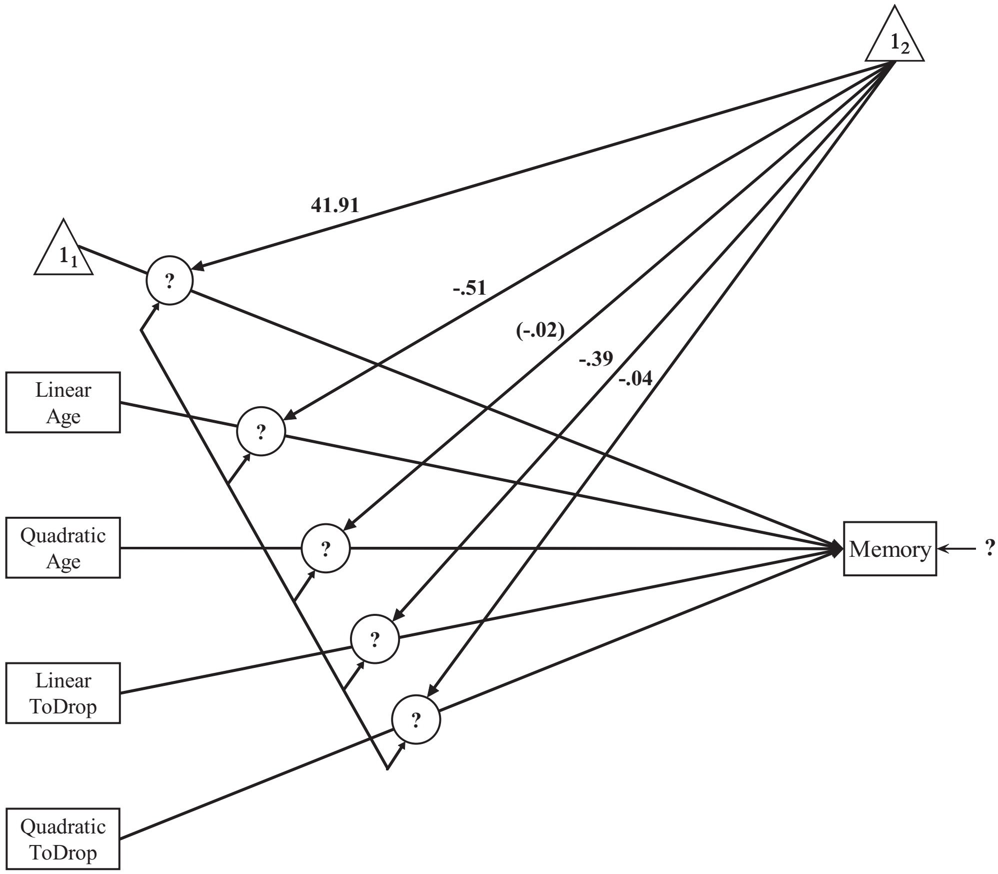

Figure 7. Path diagram applied to the random-effects regression model presented in Sliwinski et al. (2003, Table 6, Model 3a, p. 665). Parameter estimates without parentheses are significant ($p \leq .05$) and within parentheses are nonsignificant ($p > .05$). Question marks denote sample estimates for Level-1 and Level-2 random effects not reported in Sliwinski et al. ToDrop = time to drop from the study.

theoretical question of interest focused on the extent to which primary caregivers may differentially parent multiple children within the same family. The outcome was a continuous measure of positive parenting of the child. This model is presented in Figure 8.

Although this model is quite complex (eight Level-1 predictors, six Level-2 predictors, three random coefficients, one cross-level interaction), all of these details are clearly present in the diagram. There were a mixture of Level-1 and Level-2 variables that were scaled in the raw metric or were grand-mean centered; these are thus labeled in plain font and bold italic font, respectively. As before, all parameter estimates are reported; those without parentheses are statistically significant ($p \leq .05$), and those within parentheses are not ($p > .05$). The diagram highlights that, for the eight Level-1 predictors, three have random effects (intercept, age, and negative affectivity); all three random effects freely covary; all three variance estimates are significant; and only one of the three covariances is significant. Finally, only one cross-level interaction is estimated (indicated by the arrow from family size to age), and this interaction is significant.

Although this model consists of 15 measured variables, 16 fixed effects, and seven variance/covariance parameters, we could directly derive the reduced-form equation using tracing rules applied to the diagram. This example also highlights one limitation of our proposed path diagramming approach. Specifically, although the original analyses included a complex structured Level-1 residual term, this cannot be easily expressed in the diagram and would instead need to be noted in the figure caption or text. Despite this limitation, we believe the diagram helps increase the communication of these complex results to a broader audience of researchers.

Example 3: Molnar, Buka, Brennan, Holton, and Earls (2003)

Our final example focuses on a three-level model presented by Molnar et al. (2003, Table 5, Model 4, p. 92). This

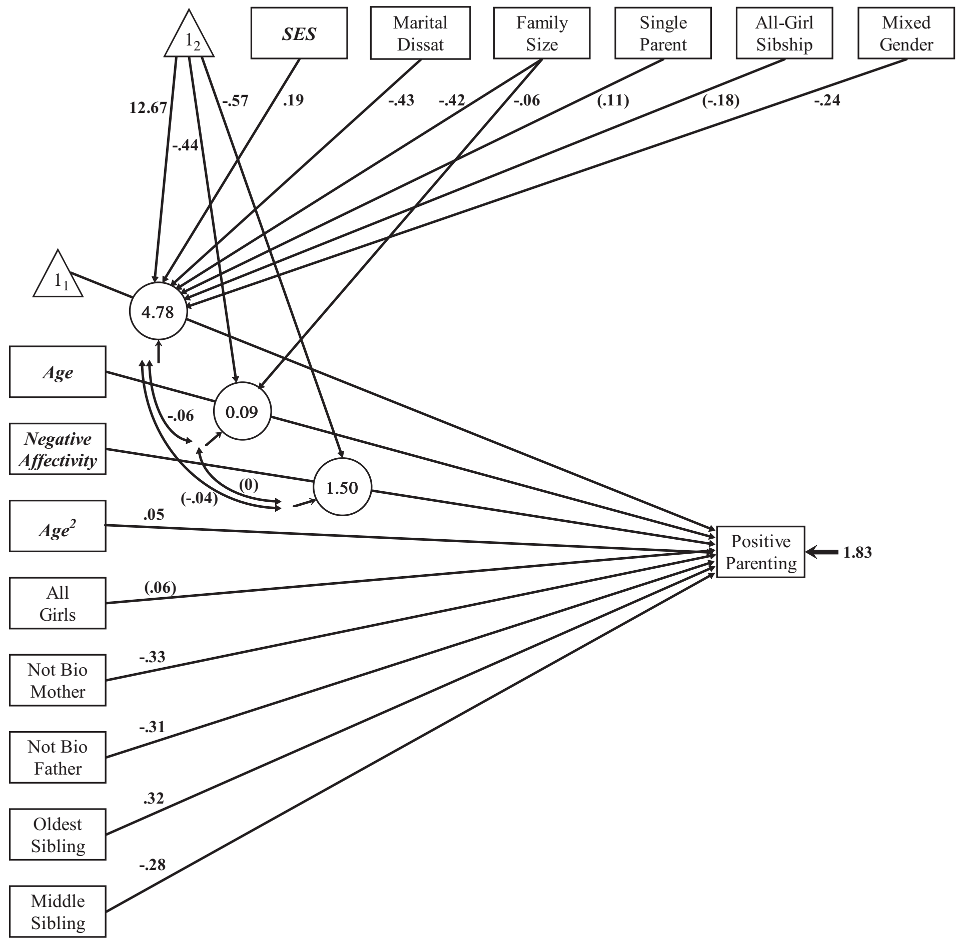

Figure 8. Path diagram applied to the two-level model presented in Jenkins et al. (2003, Table 1, Model 7, p. 104). Parameter estimates without parentheses are significant ($p \leq .05$) and within parentheses are nonsignificant ($p > .05$). The Level-1 residual has a more complex structure than can be compactly presented here. Plain font reflects raw metric; bold italic font reflects grand-mean centering. SES = socioeconomic status; Dissat = dissatisfaction; Sibship = sibling relationship; Bio = biological.

study examined the joint contribution of child, family, and community characteristics in the prediction of parent-to-child physical aggression. This is a three-level model consisting of 4,252 children nested within 3,465 families who were in turn nested within 343 neighborhood clusters. There were five Level-1 predictors (including four dummy variables to code age groups), eight Level-2 predictors (including three dummy variables to code ethnic groups), and three Level-3 predictors. The Level-1 residual was estimated, as were random intercepts at both Level 2 and Level 3. This results in a model with 16 fixed effects and three random effects. This model is presented in Figure 9.

As with the prior two examples, the path diagram allows for the explicit identification of the levels of nesting, the level at which each predictor resides, and the parameterization of both fixed and random effects. There were a mixture of Level-1, Level-2, and Level-3 variables that were scaled in the raw metric or were grand-mean centered; these are thus labeled in plain font and bold italic font, respectively. It is also clear that, because all predictors only directly influence lower order random intercept terms, this model contains strictly main effects. Further, significant effects were found for three of the five Level-1 predictors, six of the eight Level-2 predictors, one of the three Level-3 predictors, and the Level-3 intercept. Finally, because parameter estimates were not reported for either the Level-1 residual or the two random intercept effects, these are denoted in the figure by a question mark.

Conclusion

We have described a framework for graphically representing a broad class of multilevel models. To accomplish this, we have borrowed some aspects of the SEM path

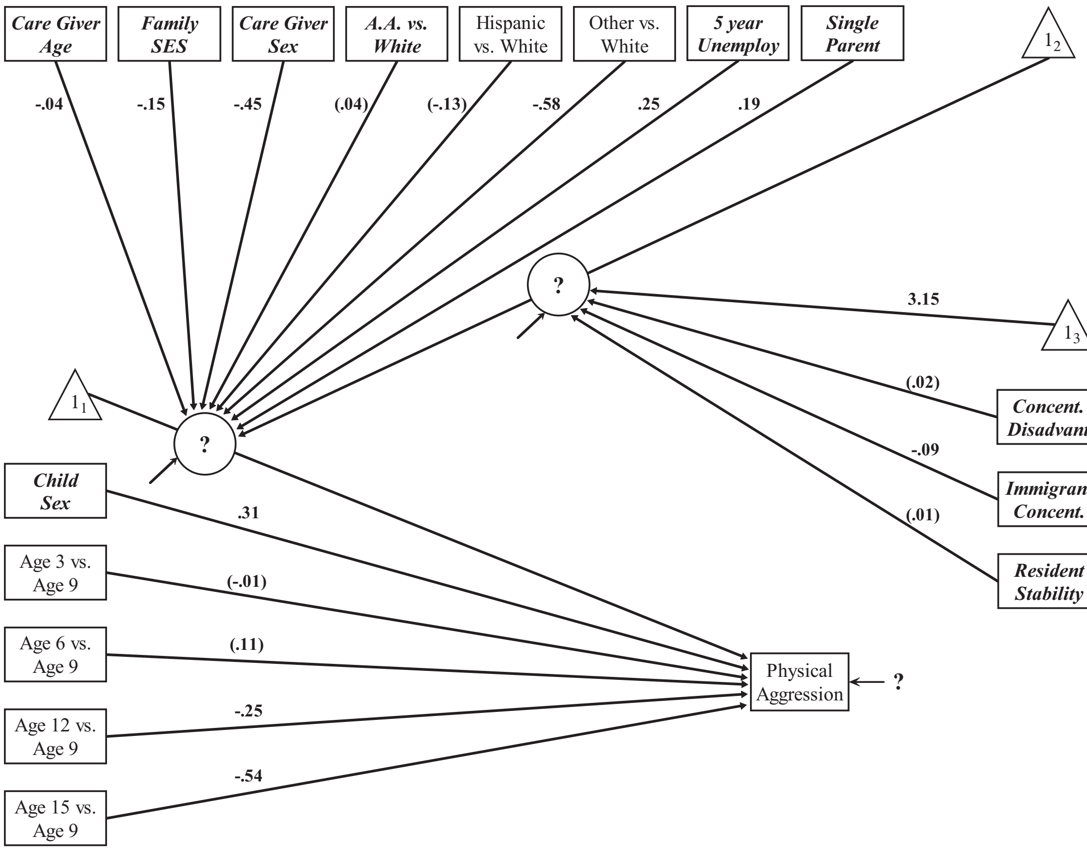

Figure 9. Path diagram applied to the three-level model presented in Molnar et al. (2003; Table 5, Model 4, p. 92). Parameter estimates without parentheses are significant ($p \leq .05$) and within parentheses are nonsignificant ($p > .05$). Question marks denote sample estimates for Level-1, Level-2, and Level-3 random effects not reported in Molnar et al. Plain font reflects raw metric; bold italic font reflects grand-mean centering. Age is in years. SES = socioeconomic status; A.A. = African American; Concent. Disadvant = concentrated disadvantage.

diagram and have proposed new components necessary to accommodate the unique characteristics of the multilevel model. We see four specific advantages of the diagramming framework: It can be used to represent a model in lieu of model equations, the diagram of the model structure is independent of any particular notation system for the model, model equations can be derived directly from the diagram, and the diagram can be used to simultaneously convey both the specific structure of the tested model and the associated empirical results. Despite what we hope are important advantages associated with the proposed diagramming strategy, there are of course several accompanying limitations.

First, as with any diagramming framework, the multilevel path diagrams naturally become increasingly cluttered when applied to highly complex models. Drawing on our experiences within a variety of research venues, we believe that the majority of empirical applications in the social sciences can be efficiently presented using the diagrams we describe here. However, it is sometimes recommended that the same set of predictor variables be used for all random effects to help protect against model misspecification (e.g., Raudenbush & Bryk, 2002, p. 272), and this strategy can result in an increasingly complex model. Under such situations a single diagram may be of less use. Second, there are several characteristics of some model parameterizations that cannot be portrayed here. Examples include complex Level-1 covariance structures and the imposition of linear or nonlinear constraints placed on two or more parameter estimates. A reasonable strategy under these conditions might be to use a path diagram and simply note such structures in the text or figure caption. Finally, there of course may be specific situations in which a tabular or equation-based presentation is preferable to the use of a path diagram. We are certainly not advocating the universal and mandatory use of diagrams; instead, we hope that a diagram might simply serve to augment other modes of communication.

There are two areas of future research that might be of much interest and utility. First, these path diagrams expand logically to incorporate cross-classified structures (e.g., Raudenbush, 1993), multivariate models for multiple outcome variables (e.g., MacCallum, Kim, Malarkey, & Kiecolt-Glaser, 1997), and multilevel models with mediating pathways (e.g., Bauer, Preacher, & Gil, 2006). Modest modifications will also allow for the depiction of multilevel generalized linear models with nonlinear link functions and nonnormal conditional distributions for the outcomes. Future work could explore how to best incorporate these expansions into the proposed framework. Second, the path diagramming framework could in principle be integrated into statistical software packages. One of the core motivating goals of applied statistics is the dissemination of advanced methods into empirical research settings. Because there are only so many hours in a given day, all people cannot be experts in all aspects of the research process. There are thus many individuals conducting important and timely research who might greatly benefit from the use of multilevel models, yet may not be familiar with using these methods in practice. Although possibly both good and bad, path diagrams have found their way into many SEM software packages, increasing the accessibility of these methods to a broader audience of researchers. The incorporation of path diagrams into multilevel modeling software might similarly increase the availability and utility of these techniques in practice. Finally, diagrams and diagram-based software can be invaluable teaching resources for students first learning about multilevel models.

We would like to conclude with a final word of warning. An anonymous reviewer was not enthusiastic about our proposed diagramming strategy given a significant concern that such a framework could serve to encourage bad work, foster mechanical thinking, and restrict creativity. Similar arguments have been made about the ubiquity of path diagrams within the SEM (see, e.g., the lyrics of David Rogosa's, 1988, *Ballad of a Casual Modeler*). We greatly appreciate the perspective offered by the reviewer, and we agree that the use of path diagrams within SEMs has not been without problems. Certainly, the availability of SEM software that accepts path diagrams as inputs has at times enabled inexperienced users to fit ill-formulated models. At the same time, we firmly believe that the advantages of model diagrams vastly outweigh their potential weaknesses. In our own experience, diagramming a model often inspires more thoughtful discussion of the model structure, its underlying assumptions, and its potential shortcomings compared to writing out the full set of equations; we have found this to be the case even among those who are already quite familiar with expressing the model in equation form. Thus, despite the potential for misuse, we hope that the careful and thoughtful application of our proposed path diagramming framework can serve to increase the understanding of multilevel models, encourage the broader use of these models in practice, and more efficiently disseminate important empirical findings to the scientific community.

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