

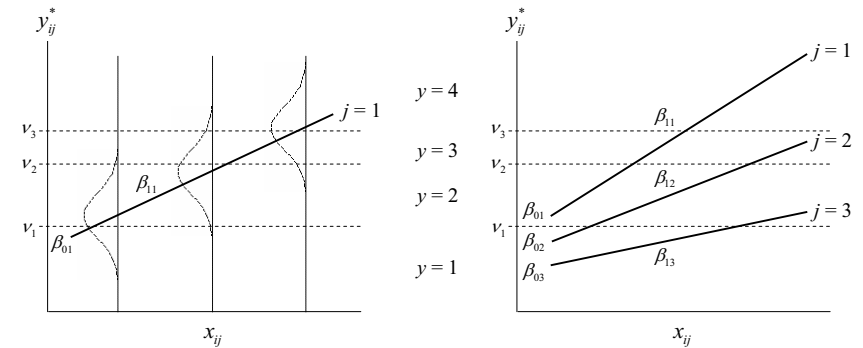
## When Bias and RMSE Don't Agree

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## Multilevel Ordered Logit Model



Level 1

$$y_{ij}^* = \beta_{0j} + \beta_{1j}x_{ij} + r_{ij}$$

$$P(y_{ij} = c | x_{ij}) = P(v_{c-1} \leq y_{ij}^* < v_c | x_{ij})$$

Level 2

$$\beta_{0j} = \gamma_{00} + \gamma_{01}w_j + u_{0j}$$

$$\beta_{1j} = \gamma_{10} + \gamma_{11}w_j + u_{1j}$$

## Simulation Study

$$P(y_{ij} = c | x_{ij}) = P(v_{c-1} \leq y_{ij}^* < v_c | x_{ij}) \quad c = 1, 2, \dots, C \quad C = \{2, 3, 5, 7\}$$

$$y_{ij}^* = \beta_{0j} + \beta_{1j}x_{ij} + r_{ij} \quad i = 1, 2, \dots, M \quad M = \{5, 10, 20\}$$

$$\beta_{0j} = \mathbf{0} + \mathbf{1}w_j + u_{0j} \quad j = 1, 2, \dots, N$$

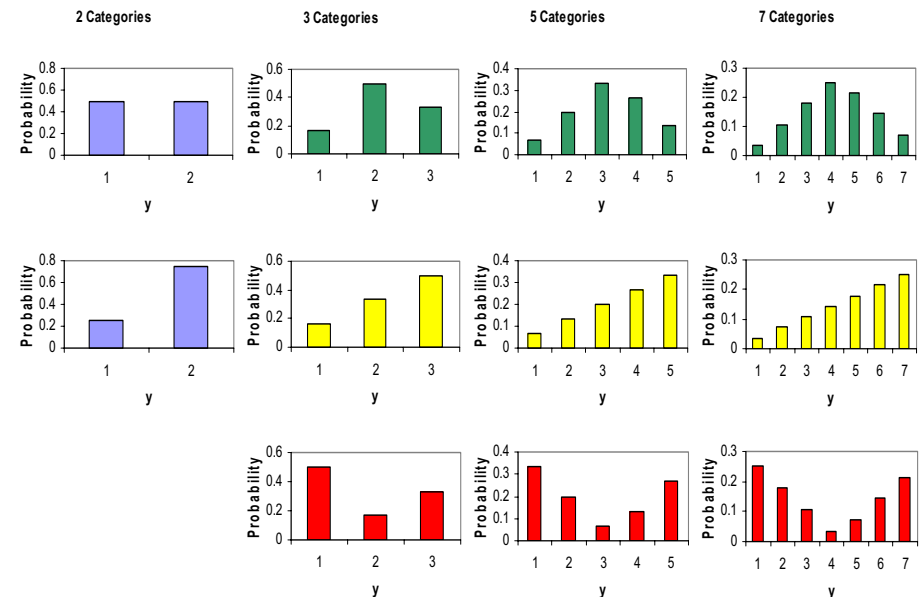
$$\beta_{1j} = \mathbf{1} + .75w_j + u_{1j} \quad N = \{25, 50, 100, 200\}$$

$$y_{ij}^* = \mathbf{0} + \mathbf{1}x_{ij} + \mathbf{1}w_j + \mathbf{.75}x_{ij}w_j + u_{0j} + u_{1j}x_{ij} + r_{ij}$$

Fixed Effects

$$V \begin{pmatrix} u_{0j} \\ u_{1j} \end{pmatrix} = \begin{pmatrix} 1.63 \\ .1 & .25 \end{pmatrix} \text{ or } \begin{pmatrix} .5 \\ .03 & .08 \end{pmatrix}$$

## Marginal Category Distributions



# Evaluating Estimators

- Bias

$$E(\hat{\theta}) - \theta$$

Relative Bias

$$\frac{E(\hat{\theta}) - \theta}{\theta}$$

- Variance

$$V(\hat{\theta})$$

Standard Deviation

$$V(\hat{\theta})^{1/2}$$

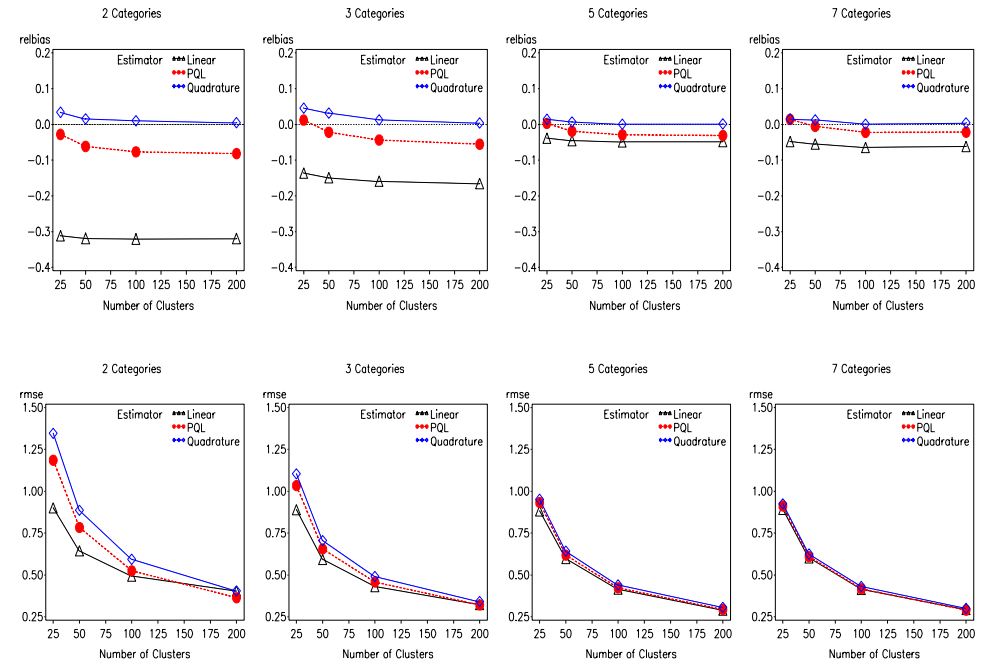
- Mean Squared Error

$$E[(\hat{\theta} - \theta)^2] = [E(\hat{\theta}) - \theta]^2 + V(\hat{\theta})$$

Root Mean Squared Error

$$E[(\hat{\theta} - \theta)^2]^{1/2}$$

## Mean Relative Bias (top) & RMSE (bottom) of Fixed Effects



## Mean Relative Bias (top) & RMSE (bottom) of Fixed Effects

