

## Detailed Simulation Results and Figures

The following set of 12 tables and Figures provide detailed results from our simulation study. The first 4 tables summarize results from the first study that examines spurious random slope variance due to nonlinearity. The tables are organized by the two primary simulation design factors: degree of curvature of the nonlinear generating model and the size of intra-class correlation coefficient. The entries are further broken down by the number of ISUs and ISU size. Analytical results for the expected values of the parameters are shown at the bottom of each table for easy reference. The remaining 8 tables are for the second simulation study where spurious cross-level interaction effect is examined. The tables are organized in a similar fashion, with the addition of a third design factor: correlation between the level-2 predictor  $W$  and the mean of the level-1 predictor  $X$ . All tables are based on 500 Monte Carlo replications.

The Figures show power curves of the likelihood ratio test for the detection of spurious random slope variance, as well as the Wald test for the detection of spurious cross-level interaction. The captions are self-explanatory.

Table 1

*Simulation Study I:  $\gamma_{20} = -0.025$  and  $ICC = 0.15$*

# ISU	ISU Size	Fixed-effects		Covariance Parameters		
		$\hat{\gamma}_{00}^*$	$\hat{\gamma}_{10}^*$	$\hat{\tau}_{00}^*$	$\hat{\tau}_{01}^*$	$\hat{\tau}_{11}^*$
50	5	9.56	1.99	1.92	0.00	0.023
	10	9.55	2.00	1.96	-0.01	0.015
	25	9.56	2.00	1.94	0.00	0.010
	50	9.59	2.00	1.96	0.00	0.009
100	5	9.55	2.00	1.87	0.00	0.019
	10	9.55	2.00	1.95	0.00	0.014
	25	9.55	2.00	1.95	0.00	0.010
	50	9.57	2.00	1.98	0.00	0.009
200	5	9.53	2.00	1.91	0.00	0.020
	10	9.53	2.00	1.96	0.00	0.014
	25	9.55	2.00	1.99	0.00	0.010
	50	9.57	2.00	1.98	0.00	0.009
300	5	9.54	2.00	1.95	0.00	0.019
	10	9.54	2.00	1.95	0.00	0.014
	25	9.55	2.00	1.97	0.00	0.010
	50	9.57	2.00	1.99	0.00	0.009

*Note.* The entries in the table are averaged over 500 converged replications; According to the generating values and Appendix A,  $\gamma_{00}^* = 9.65$ ,  $\gamma_{10}^* = 2$ ,  $\tau_{00}^* = 2.01$ ,  $\tau_{01}^* = 0$ ,  $\tau_{11}^* = 0.008$ .

Table 2

*Simulation Study I:  $\gamma_{20} = -0.025$  and ICC = 0.50*

# ISU	ISU Size	Fixed-effects		Covariance Parameters		
		$\hat{\gamma}_{00}^*$	$\hat{\gamma}_{10}^*$	$\hat{\tau}_{00}^*$	$\hat{\tau}_{01}^*$	$\hat{\tau}_{11}^*$
50	5	9.61	2.00	1.77	0.00	0.029
	10	9.62	2.00	1.78	0.00	0.025
	25	9.68	2.00	1.88	0.00	0.023
	50	9.78	2.00	1.95	0.00	0.023
100	5	9.59	2.00	1.80	0.00	0.027
	10	9.62	2.00	1.82	0.00	0.024
	25	9.68	2.00	1.87	0.00	0.023
	50	9.77	2.00	1.95	0.00	0.023
200	5	9.58	2.00	1.83	0.00	0.027
	10	9.61	2.00	1.89	0.00	0.024
	25	9.69	2.00	1.91	0.00	0.024
	50	9.77	2.00	1.94	0.00	0.024
300	5	9.58	2.00	1.83	0.00	0.027
	10	9.62	2.00	1.87	0.00	0.025
	25	9.69	2.00	1.92	0.00	0.023
	50	9.77	2.00	1.96	0.00	0.024

*Note.* The entries in the table are averaged over 500 converged replications; According to the generating values and Appendix A,  $\gamma_{00}^* = 10$ ,  $\gamma_{10}^* = 2$ ,  $\tau_{00}^* = 2.125$ ,  $\tau_{01}^* = 0$ ,  $\tau_{11}^* = 0.025$ .

Table 3

*Simulation Study I:  $\gamma_{20} = -0.10$  and  $ICC = 0.15$*

# ISU	ISU Size	Fixed-effects		Covariance Parameters		
		$\hat{\gamma}_{00}^*$	$\hat{\gamma}_{10}^*$	$\hat{\tau}_{00}^*$	$\hat{\tau}_{01}^*$	$\hat{\tau}_{11}^*$
50	5	8.63	2.00	1.88	0.00	0.220
	10	8.59	2.01	1.98	0.01	0.180
	25	8.57	2.00	2.06	0.00	0.150
	50	8.58	2.00	2.11	0.00	0.136
100	5	8.63	2.00	1.93	0.00	0.220
	10	8.57	2.00	1.98	0.00	0.182
	25	8.58	2.00	2.08	0.00	0.149
	50	8.56	2.00	2.11	0.00	0.135
200	5	8.62	2.00	1.87	0.00	0.224
	10	8.58	1.99	1.95	0.00	0.186
	25	8.58	2.00	2.10	0.00	0.153
	50	8.59	2.00	2.12	0.00	0.138
300	5	8.60	2.00	1.88	0.00	0.225
	10	8.57	2.00	2.02	0.00	0.185
	25	8.57	2.00	2.07	0.00	0.153
	50	8.58	2.00	2.11	0.00	0.138

*Note.* The entries in the table are averaged over 500 converged replications; According to the generating values and Appendix A,  $\gamma_{00}^* = 8.6$ ,  $\gamma_{10}^* = 2$ ,  $\tau_{00}^* = 2.18$ ,  $\tau_{01}^* = 0$ ,  $\tau_{11}^* = 0.12$ .

Table 4

*Simulation Study I:  $\gamma_{20} = -0.10$  and  $ICC = 0.50$*

# ISU	ISU Size	Fixed-effects		Covariance Parameters		
		$\hat{\gamma}_{00}^*$	$\hat{\gamma}_{10}^*$	$\hat{\tau}_{00}^*$	$\hat{\tau}_{01}^*$	$\hat{\tau}_{11}^*$
50	5	9.17	1.98	1.58	-0.04	0.312
	10	9.38	2.00	1.97	-0.02	0.310
	25	9.66	1.99	2.68	-0.01	0.340
	50	9.83	2.00	3.13	-0.01	0.368
100	5	9.15	2.00	1.52	-0.02	0.310
	10	9.38	2.00	2.00	0.00	0.316
	25	9.65	2.00	2.63	0.01	0.345
	50	9.82	2.00	3.10	-0.01	0.367
200	5	9.13	2.00	1.49	0.01	0.309
	10	9.37	2.00	1.96	-0.01	0.318
	25	9.67	2.00	2.65	0.01	0.349
	50	9.82	2.00	3.06	-0.01	0.368
300	5	9.15	2.00	1.45	0.01	0.310
	10	9.38	2.00	1.96	-0.01	0.320
	25	9.66	2.00	2.60	0.00	0.348
	50	9.81	2.00	3.09	-0.01	0.369

*Note.* The entries in the table are averaged over 500 converged replications; According to the generating values and Appendix A,  $\gamma_{00}^* = 10$ ,  $\gamma_{10}^* = 2$ ,  $\tau_{00}^* = 4$ ,  $\tau_{01}^* = 0$ ,  $\tau_{11}^* = 0.40$ .

Table 5

*Simulation Study II:  $\gamma_{20} = -0.025$ ,  $ICC = 0.15$ , and  $\rho = 0.6$*

# ISU	ISU Size	Fixed-effects				Slope Variance
		$\hat{\gamma}_{00}^*$	$\hat{\gamma}_{10}^*$	$\hat{\gamma}_{01}^*$	$\hat{\gamma}_{11}^*$	$\hat{\tau}_{11}^*$
50	5	9.58	2.00	1.00	-0.011	0.018
	10	9.58	2.00	1.00	-0.012	0.012
	25	9.56	2.00	1.01	-0.012	0.007
	50	9.61	2.00	1.00	-0.013	0.006
100	5	9.58	2.00	1.00	-0.011	0.017
	10	9.59	2.00	1.00	-0.013	0.011
	25	9.59	2.00	1.00	-0.012	0.007
	50	9.60	2.00	1.00	-0.013	0.006
200	5	9.59	2.00	1.00	-0.012	0.016
	10	9.59	2.00	1.00	-0.012	0.011
	25	9.58	2.00	1.00	-0.012	0.007
	50	9.59	2.00	1.00	-0.012	0.006
300	5	9.57	2.00	1.00	-0.012	0.015
	10	9.58	2.00	1.00	-0.012	0.011
	25	9.59	2.00	1.00	-0.013	0.007
	50	9.59	2.00	1.00	-0.012	0.006

*Note.* The entries in the table are averaged over 500 converged replications; According to the generating values and Appendix B,  $\gamma_{00}^* = 9.65$ ,  $\gamma_{10}^* = 2$ ,  $\gamma_{01}^* = 1$ ,  $\gamma_{11}^* = -0.0126$ ,  $\tau_{11}^* = 0.0048$ .

Table 6

*Simulation Study II:  $\gamma_{20} = -0.025$ ,  $ICC = 0.50$ , and  $\rho = 0.6$*

# ISU	ISU Size	Fixed-effects				Slope Variance
		$\hat{\gamma}_{00}^*$	$\hat{\gamma}_{10}^*$	$\hat{\gamma}_{01}^*$	$\hat{\gamma}_{11}^*$	$\hat{\tau}_{11}^*$
50	5	9.70	2.00	1.00	-0.025	0.020
	10	9.72	2.00	1.00	-0.027	0.017
	25	9.77	2.00	1.00	-0.028	0.014
	50	9.81	2.00	1.00	-0.029	0.014
100	5	9.71	2.00	1.00	-0.026	0.019
	10	9.72	2.00	1.00	-0.026	0.015
	25	9.77	2.00	1.00	-0.028	0.014
	50	9.80	2.00	1.00	-0.029	0.015
200	5	9.70	2.00	1.00	-0.025	0.016
	10	9.71	2.00	1.00	-0.027	0.015
	25	9.75	2.00	1.00	-0.027	0.015
	50	9.81	2.00	1.00	-0.029	0.015
300	5	9.69	2.00	1.00	-0.026	0.017
	10	9.70	2.00	1.00	-0.026	0.015
	25	9.76	2.00	1.00	-0.028	0.015
	50	9.81	2.00	1.00	-0.029	0.015

*Note.* The entries in the table are averaged over 500 converged replications; According to the generating values and Appendix B,  $\gamma_{00}^* = 9.65$ ,  $\gamma_{10}^* = 2$ ,  $\gamma_{01}^* = 1$ ,  $\gamma_{11}^* = -0.03$ ,  $\tau_{11}^* = 0.016$ .

Table 7

*Simulation Study II:  $\gamma_{20} = -0.10$ ,  $ICC = 0.15$ , and  $\rho = 0.6$*

# ISU	ISU Size	Fixed-effects				Slope Variance
		$\hat{\gamma}_{00}^*$	$\hat{\gamma}_{10}^*$	$\hat{\gamma}_{01}^*$	$\hat{\gamma}_{11}^*$	$\hat{\tau}_{11}^*$
50	5	8.66	2.00	1.00	-0.044	0.178
	10	8.61	2.00	1.00	-0.046	0.141
	25	8.57	2.00	1.00	-0.049	0.107
	50	8.57	2.00	1.00	-0.049	0.094
100	5	8.66	2.00	1.00	-0.044	0.187
	10	8.62	2.00	1.00	-0.046	0.149
	25	8.58	2.00	1.00	-0.049	0.110
	50	8.58	2.00	1.00	-0.049	0.096
200	5	8.66	2.00	1.00	-0.043	0.191
	10	8.61	2.00	1.00	-0.046	0.152
	25	8.58	2.00	1.00	-0.048	0.112
	50	8.58	2.00	1.00	-0.050	0.096
300	5	8.65	2.00	1.00	-0.043	0.191
	10	8.60	2.00	1.00	-0.046	0.150
	25	8.59	2.00	1.00	-0.048	0.113
	50	8.58	2.00	1.00	-0.049	0.097

*Note.* The entries in the table are averaged over 500 converged replications; According to the generating values and Appendix B,  $\gamma_{00}^* = 8.6$ ,  $\gamma_{10}^* = 2$ ,  $\gamma_{01}^* = 1$ ,  $\gamma_{11}^* = -0.05$ ,  $\tau_{11}^* = 0.0768$ .



Table 8

*Simulation Study II:  $\gamma_{20} = -0.10$ ,  $ICC = 0.50$ , and  $\rho = 0.6$*

# ISU	ISU Size	Fixed-effects				Slope Variance
		$\hat{\gamma}_{00}^*$	$\hat{\gamma}_{10}^*$	$\hat{\gamma}_{01}^*$	$\hat{\gamma}_{11}^*$	$\hat{\tau}_{11}^*$
50	5	9.22	2.01	1.00	-0.094	0.199
	10	9.42	2.00	1.00	-0.102	0.205
	25	9.67	2.00	1.00	-0.109	0.220
	50	9.82	2.00	1.00	-0.115	0.234
100	5	9.25	2.00	1.00	-0.093	0.212
	10	9.43	2.00	1.00	-0.101	0.210
	25	9.68	2.00	1.00	-0.110	0.227
	50	9.81	2.00	1.00	-0.114	0.238
200	5	9.24	2.00	1.00	-0.094	0.217
	10	9.42	2.00	1.00	-0.100	0.215
	25	9.67	2.00	1.00	-0.108	0.231
	50	9.82	2.00	1.00	-0.113	0.241
300	5	9.23	2.00	1.00	-0.093	0.214
	10	9.41	2.00	1.00	-0.099	0.215
	25	9.67	2.00	1.00	-0.109	0.231
	50	9.82	2.00	1.00	-0.113	0.242

*Note.* The entries in the table are averaged over 500 converged replications; According to the generating values and Appendix B,  $\gamma_{00}^* = 10$ ,  $\gamma_{10}^* = 2$ ,  $\gamma_{01}^* = 1$ ,  $\gamma_{11}^* = -0.12$ ,  $\tau_{11}^* = 0.265$ .

Table 9

*Simulation Study II:  $\gamma_{20} = -0.025$ ,  $ICC = 0.15$ , and  $\rho = 0.3$*

# ISU	ISU Size	Fixed-effects				Slope Variance
		$\hat{\gamma}_{00}^*$	$\hat{\gamma}_{10}^*$	$\hat{\gamma}_{01}^*$	$\hat{\gamma}_{11}^*$	$\hat{\tau}_{11}^*$
50	5	9.57	2.00	1.00	-0.005	0.019
	10	9.56	2.00	1.00	-0.007	0.013
	25	9.57	2.00	1.00	-0.006	0.009
	50	9.58	2.00	1.00	-0.006	0.007
100	5	9.56	2.00	1.00	-0.006	0.018
	10	9.55	2.00	1.00	-0.006	0.013
	25	9.55	2.00	1.00	-0.007	0.009
	50	9.58	2.00	1.00	-0.006	0.008
200	5	9.56	2.00	1.00	-0.006	0.018
	10	9.55	2.00	1.00	-0.006	0.012
	25	9.56	2.00	1.00	-0.006	0.009
	50	9.58	2.00	1.00	-0.006	0.008
300	5	9.55	2.00	1.00	-0.006	0.018
	10	9.55	2.00	1.01	-0.006	0.013
	25	9.56	2.00	1.00	-0.006	0.009
	50	9.58	2.00	1.00	-0.006	0.008

*Note.* The entries in the table are averaged over 500 converged replications; According to the generating values and Appendix B,  $\gamma_{00}^* = 9.65$ ,  $\gamma_{10}^* = 2$ ,  $\gamma_{01}^* = 1$ ,  $\gamma_{11}^* = -0.0063$ ,  $\tau_{11}^* = 0.0068$ .

Table 10

*Simulation Study II:  $\gamma_{20} = -0.025$ ,  $ICC = 0.50$ , and  $\rho = 0.3$*

# ISU	ISU Size	Fixed-effects				Slope Variance
		$\hat{\gamma}_{00}^*$	$\hat{\gamma}_{10}^*$	$\hat{\gamma}_{01}^*$	$\hat{\gamma}_{11}^*$	$\hat{\tau}_{11}^*$
50	5	9.62	2.00	1.00	-0.013	0.024
	10	9.64	2.00	1.00	-0.013	0.020
	25	9.70	2.00	1.00	-0.014	0.019
	50	9.78	2.00	1.00	-0.014	0.020
100	5	9.60	2.00	1.00	-0.014	0.023
	10	9.65	2.00	1.00	-0.014	0.021
	25	9.71	2.00	1.00	-0.013	0.020
	50	9.77	2.00	1.00	-0.014	0.021
200	5	9.62	2.00	1.00	-0.013	0.024
	10	9.64	2.00	1.00	-0.013	0.022
	25	9.70	2.00	1.00	-0.014	0.021
	50	9.77	2.00	1.00	-0.014	0.021
300	5	9.61	2.00	1.00	-0.013	0.023
	10	9.65	2.00	1.00	-0.014	0.022
	25	9.70	2.00	1.00	-0.014	0.021
	50	9.77	2.00	1.00	-0.014	0.021

*Note.* The entries in the table are averaged over 500 converged replications; According to the generating values and Appendix B,  $\gamma_{00}^* = 10$ ,  $\gamma_{10}^* = 2$ ,  $\gamma_{01}^* = 1$ ,  $\gamma_{11}^* = -0.015$ ,  $\tau_{11}^* = 0.02275$ .

Table 11

*Simulation Study II:  $\gamma_{20} = -0.10$ ,  $ICC = 0.15$ , and  $\rho = 0.3$*

# ISU	ISU Size	Fixed-effects				Slope Variance
		$\hat{\gamma}_{00}^*$	$\hat{\gamma}_{10}^*$	$\hat{\gamma}_{01}^*$	$\hat{\gamma}_{11}^*$	$\hat{\tau}_{11}^*$
50	5	8.63	1.99	1.01	-0.025	0.204
	10	8.56	2.01	1.00	-0.022	0.169
	25	8.58	2.00	1.00	-0.025	0.138
	50	8.57	2.00	1.00	-0.025	0.122
100	5	8.61	2.00	1.00	-0.020	0.212
	10	8.58	2.00	1.00	-0.023	0.169
	25	8.57	2.00	1.00	-0.025	0.140
	50	8.59	2.00	1.00	-0.025	0.125
200	5	8.63	2.00	1.00	-0.021	0.215
	10	8.59	2.00	1.00	-0.023	0.174
	25	8.57	2.00	1.00	-0.024	0.143
	50	8.57	2.00	1.00	-0.024	0.126
300	5	8.62	2.00	1.00	-0.022	0.215
	10	8.58	2.00	1.00	-0.023	0.174
	25	8.57	2.00	1.00	-0.024	0.143
	50	8.58	2.00	1.00	-0.025	0.127

*Note.* The entries in the table are averaged over 500 converged replications; According to the generating values and Appendix B,  $\gamma_{00}^* = 8.6$ ,  $\gamma_{10}^* = 2$ ,  $\gamma_{01}^* = 1$ ,  $\gamma_{11}^* = -0.0252$ ,  $\tau_{11}^* = 0.1092$ .

Table 12

*Simulation Study II:  $\gamma_{20} = -0.10$ ,  $ICC = 0.50$ , and  $\rho = 0.3$*

# ISU	ISU Size	Fixed-effects				Slope Variance
		$\hat{\gamma}_{00}^*$	$\hat{\gamma}_{10}^*$	$\hat{\gamma}_{01}^*$	$\hat{\gamma}_{11}^*$	$\hat{\tau}_{11}^*$
50	5	9.19	2.00	1.00	-0.049	0.279
	10	9.36	2.00	1.00	-0.051	0.275
	25	9.68	2.00	0.99	-0.054	0.310
	50	9.82	2.00	1.00	-0.057	0.326
100	5	9.19	1.99	1.00	-0.048	0.283
	10	9.39	2.00	1.00	-0.049	0.290
	25	9.66	2.00	1.00	-0.054	0.314
	50	9.82	1.99	1.00	-0.056	0.333
200	5	9.17	2.00	1.00	-0.048	0.285
	10	9.38	2.00	1.00	-0.049	0.289
	25	9.66	2.00	1.00	-0.054	0.315
	50	9.82	2.00	1.00	-0.058	0.337
300	5	9.17	2.00	1.00	-0.048	0.286
	10	9.39	2.00	1.00	-0.051	0.292
	25	9.66	2.00	1.00	-0.054	0.316
	50	9.82	2.00	1.00	-0.058	0.336

*Note.* The entries in the table are averaged over 500 converged replications; According to the generating values and Appendix B,  $\gamma_{00}^* = 10$ ,  $\gamma_{10}^* = 2$ ,  $\gamma_{01}^* = 1$ ,  $\gamma_{11}^* = -0.06$ ,  $\tau_{11}^* = 0.364$ .

## Figure Captions

*Figure 1.* Power of the Likelihood Ratio Test of the Random Slope

*Figure 2.* Power of the Wald Test of the Cross-level Interaction ( $\gamma_{20} = -0.025$ )

*Figure 3.* Power of the Wald Test of the Cross-level Interaction ( $\gamma_{20} = -0.10$ )







