Multilevel Models for Ordinal Outcomes in Longitudinal and Clustered Data

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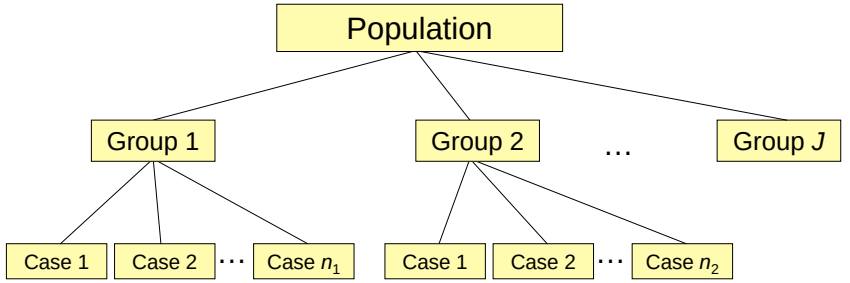
Outline

- Multilevel models: When and Why
- Peculiarities of ordinal data
- Multilevel models for ordinal data
- Evaluate the performance of three methods of estimation for multilevel models with ordinal data

Multilevel Models: When and Why

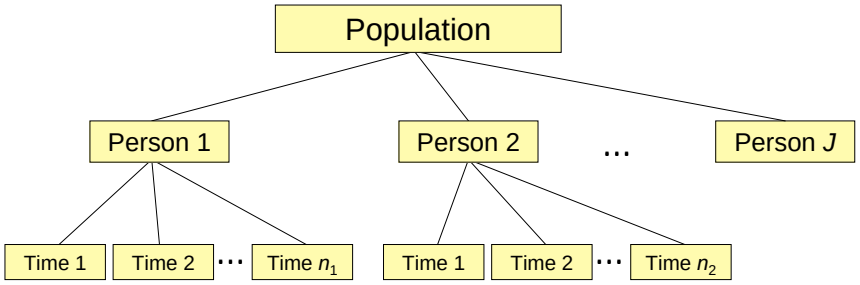
Hierarchically Clustered Data

- Psychologists often observe individuals nested within groups
  - Students within classrooms
  - Adolescents within peer groups
  - Clients within therapeutic groups



Longitudinal Data

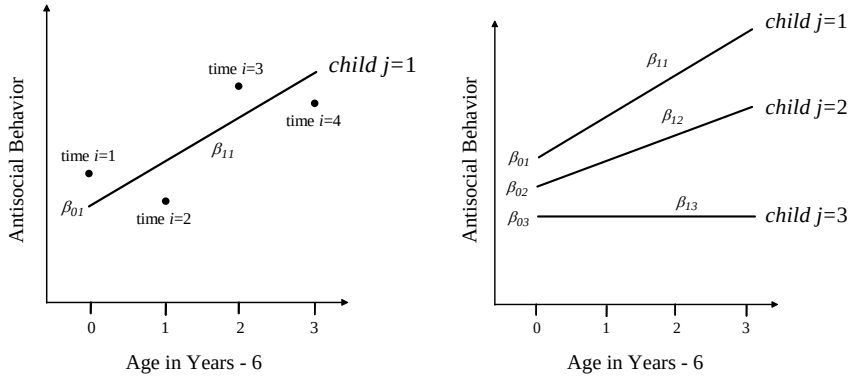
- Longitudinal designs have a similar structure
  - Changes in depression as a function of treatment
  - Increasing drowsiness of audience members during my talk



Dependence in Hierarchical and Logitudinal Data

- Dependence:
  - Individuals who belong to the same group are likely to be similar to one another.
  - Repeated measures tend to be correlated over time.
- Problem:
  - Most statistical models assume independence of observations.
- Solution:
  - Multilevel models provide a means to explicitly model and evaluate potential sources of dependence.

Example: Changes in Antisocial Behavior Over Development



Level 1

$$\text{Anti}_{ij} = \beta_{0j} + \beta_{1j}\text{Age}_{ij} + r_{ij}$$

Level 2

$$\beta_{0j} = \gamma_{00} + u_{0j}$$
$$\beta_{1j} = \gamma_{10} + u_{1j}$$

Multilevel Linear Models

- Multilevel linear models for continuous data are now well-established
  - More and more articles presenting multilevel linear models (or HLMs, mixed models)
  - Gold standard methods of estimation (REML and FIML).
- Multilevel models for discrete (e.g., ordinal) outcomes are infrequently used and lack gold standard methods of estimation.

Peculiarities of Ordinal Data

(Why your Likert Scale isn't Continuous)

Examples of Ordinal Data

How often does your child hit other children?

0 = never
1 = sometimes
2 = often

How often did you have five or more drinks of beer, wine, wine cooler, or hard liquor at one time within the past year?

1 = never
2 = once or twice
3 = more than once or twice but less than once a month
4 = one to two times a month
5 = more than twice a month

I get along well with others at work

Strongly Agree				Strongly Disagree
1	2	3	4	5

Peculiarities of Ordinal Data

- Ordinal data is discrete
- Ordinal data has upper and lower bounds
- Categories not separated by known distances
- Ordinal data often does not obey linear relationships

Strategies for Analyzing Ordinal Outcomes

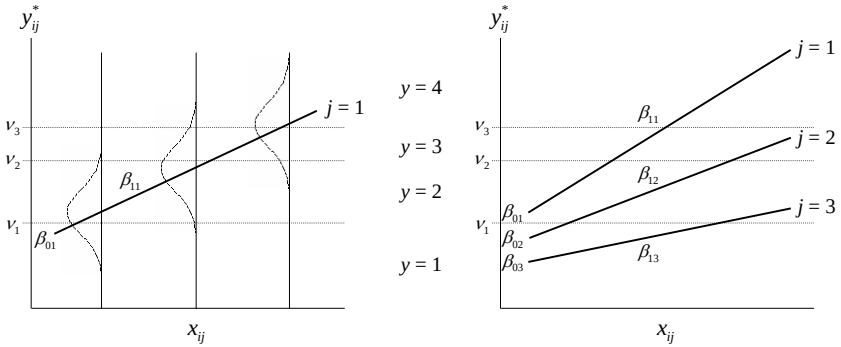
- Ad hoc: 5+ category outcomes often analyzed as if interval-level.
  - Sometimes noted that this is advisable only if the category distribution looks somewhat normal.
- Statistically Better Justified: Use a ordered logit model (proportional odds model) or ordered probit model

Multilevel Models for Ordinal Data

Multilevel Models for Ordinal Outcomes

- Latent Variable Model
  - View ordinal y as a coarse representation of an underlying continuous latent y^*
  - Posit a threshold model linking y to y^*
  - Write linear model for y^*
  - Assume an error distribution for y^* (logistic or normal)

Multilevel Ordered Logit/Probit Model



Level 1

$$y_{ij}^* = \beta_{0j} + \beta_{1j} x_{ij} + r_{ij}$$

$$P(y_{ij} = c \mid x_{ij}) = P(\nu_{c-1} \le y_{ij}^* < \nu_c \mid x_{ij})$$

Level 2

$$\beta_{0j} = \gamma_{00} + u_{0j}$$

$$\beta_{1j} = \gamma_{10} + u_{1j}$$

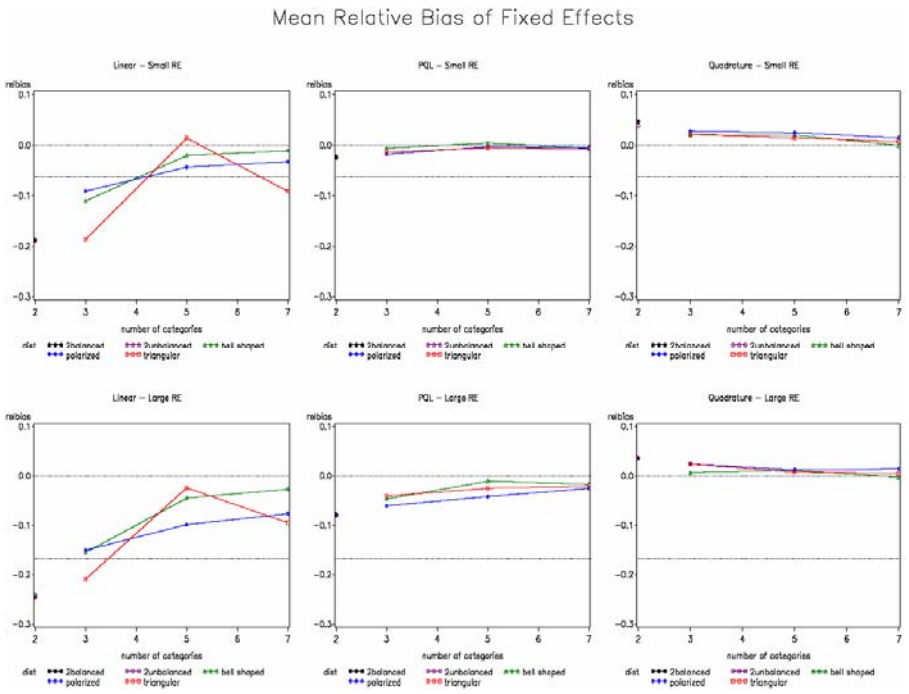
The Problem of Estimation

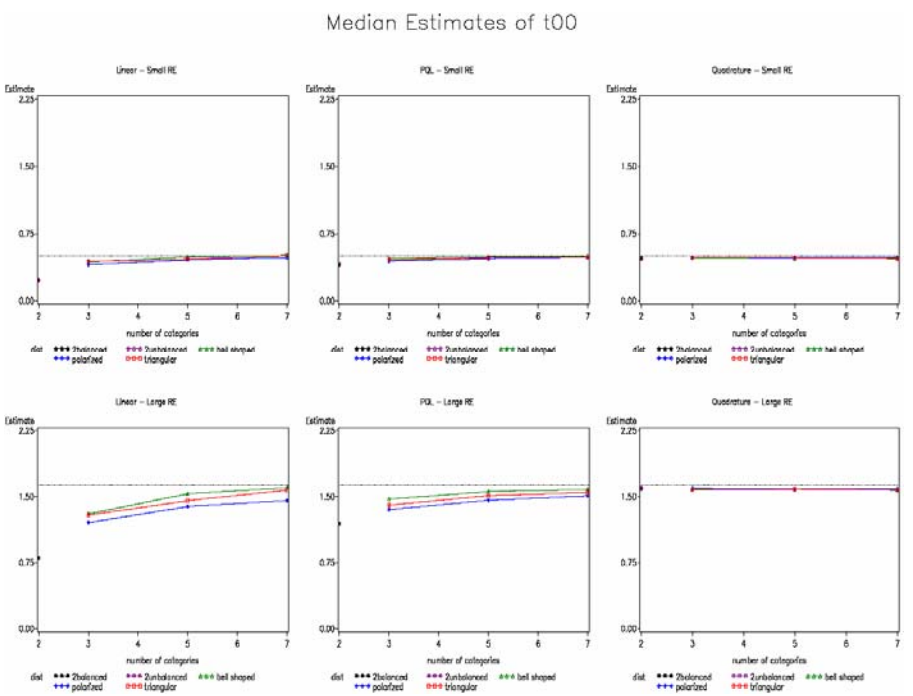
- Penalized Quasi-Likelihood (PQL)
- Numerical Integration by Adaptive Quadrature
- Many other estimators: MQL, Laplace, MCMC, etc.
- Most typical strategy: ignore the fact that the data is ordinal, fit a multilevel linear model, and hope for the best.

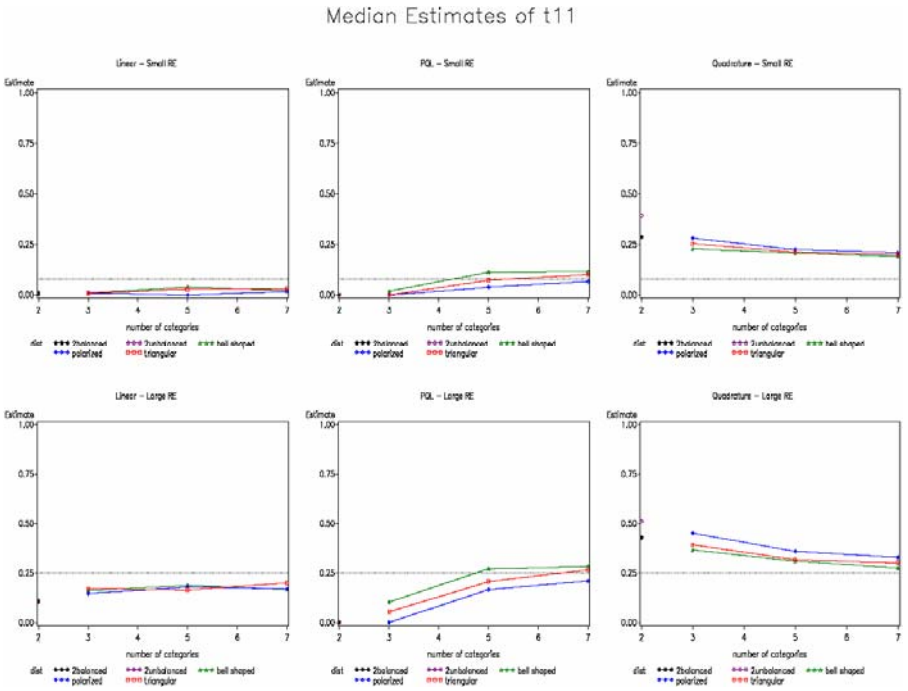
Comparing Strategies for Fitting Multilevel Models with Ordinal Outcomes

Simulation Study

- Number of categories = 2, 3, 5, or 7
- Marginal Category Distribution
  - Binary data: balanced (50:50) or unbalanced (25:75)
  - Others: Bell-Shaped, Triangular, or Polarized shapes
- Number of clusters = 25, 50, 100 or 200
- Number of individuals within cluster = 5, 10 or 20
- Small v. large variances for random effects (random intercept and random slope)
- Fit data with linear model (SAS PROC MIXED), PQL (SAS PROC GLIMMIX) and adaptive quadrature (Mplus)







Provisional Recommendations

- Perhaps best to avoid fitting multilevel linear models to ordinal data given potentially large bias and sensitivity to category distributions.
  - May produce acceptable results with 7 categories
- Quadrature preferred to PQL on asymptotics, but may not do well when few categories + small # of clusters and/or small cluster size.
- Bias in PQL estimates is greatly reduced with more categories. Minimal bias with 5+ categories.
- More information is better!!!
  - More categories (w/ linear, pql, quad)
  - More clusters (w/ quad)
  - Bigger clusters (w/ pql, quad)