An Evaluative Comparison of Random Coefficient Growth Models for Individual Development

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Association for Psychological Science

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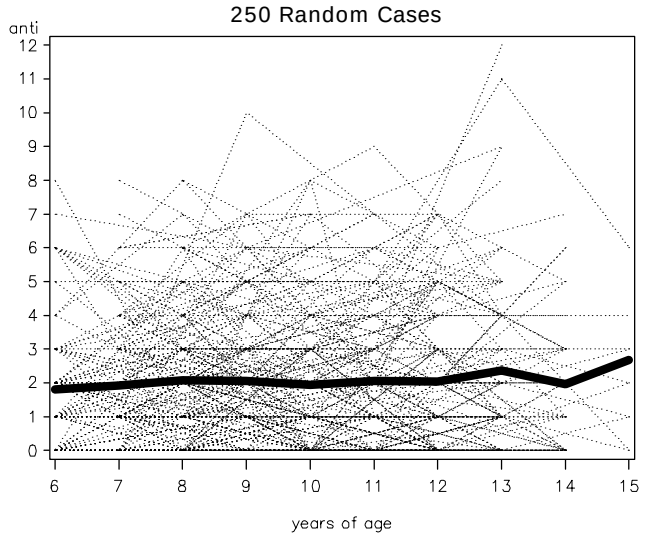
Overview of Talk

- Motivating example
- Definition of Random Coefficient Growth Model (RCGM)
- Three basic types of RCGMs
- Controversies in model selection and interpretation

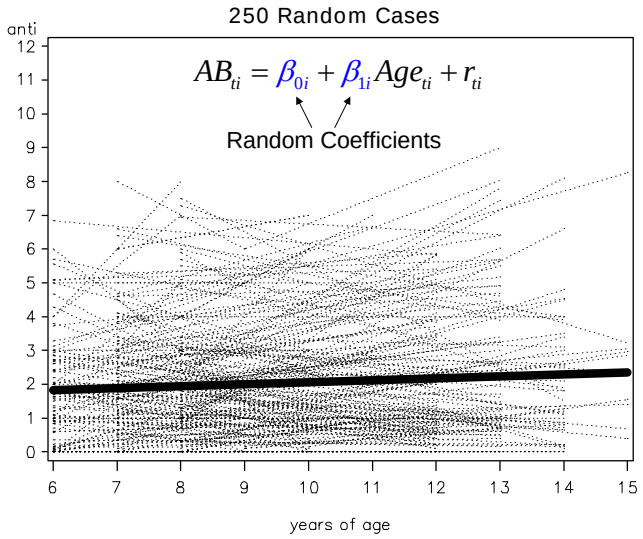
Motivating Example: Antisocial Behavior (AB)

- Concerns:
  - What do trajectories of change in AB look like from age 6 to age 15?
  - Are there sex-differences in these developmental trends?
  - Do supportive home environments protect against increases in AB?
  - Can AB be predicted by poor academic performance?
- Sample:
  - 894 children assessed biennially from 1986 to 1992 as part of the NLSY-Child Sample.
  - Between 6 and 8 years old in 1986.
- Measures:
  - Antisocial Behavior: Sum of 6 items from BPI
  - Early Home Environment: HOME-SF cognitive and emotional support scores from first assessment
  - Academic Performance: PIAT Math scores

Raw Data



The Idea Behind RCGMs



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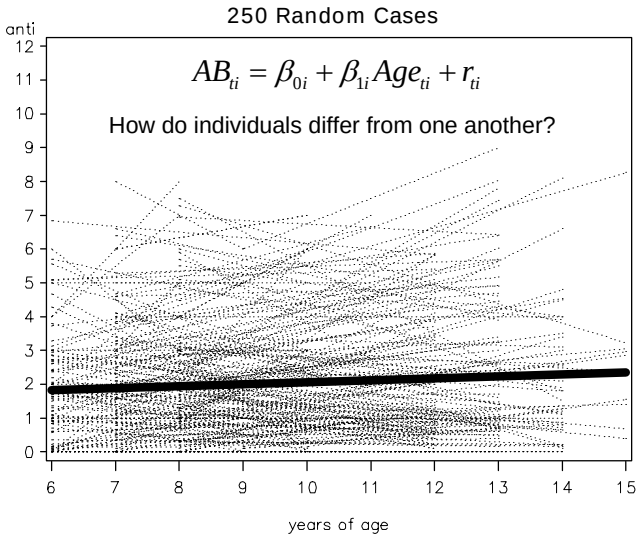
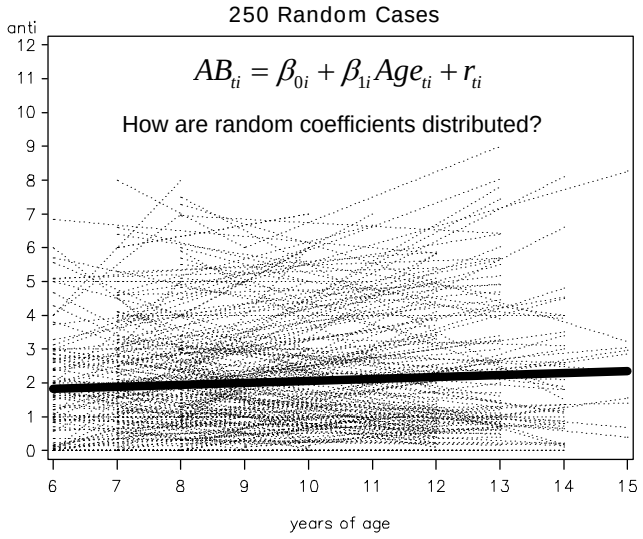


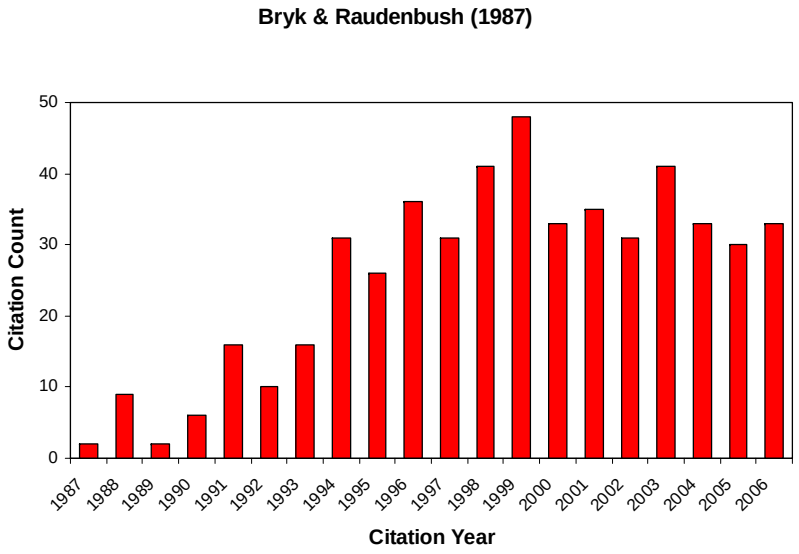
The Idea Behind RCGMs



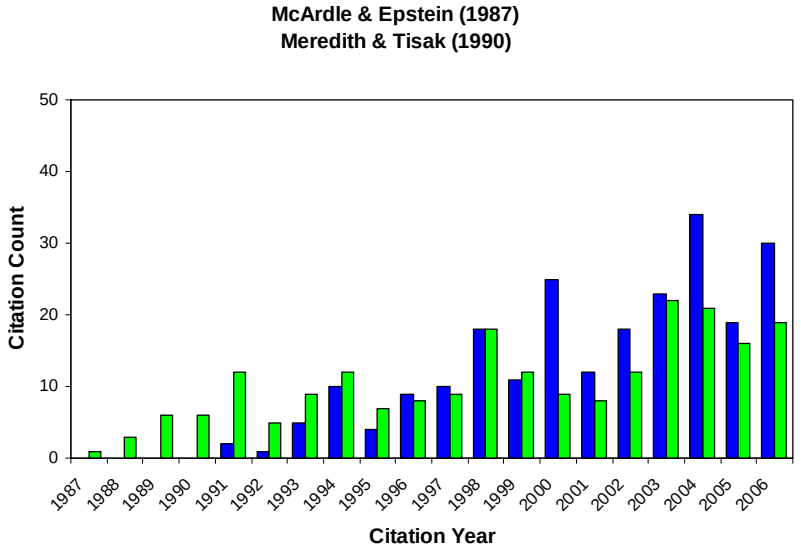
Three Types of RCGMs

1. RCN: Models that assume the random coefficients are normally distributed (conditional on predictors)
   - Latent curve/trajectory/growth models, HLM, multilevel growth models, mixed-effects models

Trends in Use of RCN Models (HLMs)



Trends in Use of RCN Models (LCMs)



RCN

- Within-Person Model:

$$AB_{ti} = \beta_{0i} + \beta_{1i}\left(Age_{ti} - 6\right) + r_{ti} \qquad r_{ti} \sim N\left(0, \sigma^2\right)$$

- Between-Person Model:

$$\beta_{0i} = \gamma_{00} + u_{0i} \qquad \begin{pmatrix} u_{0i} \\ u_{1i} \end{pmatrix} \sim N\left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \tau_{00} & \\ \tau_{10} & \tau_{11} \end{pmatrix}\right]$$
$$\beta_{1i} = \gamma_{10} + u_{1i}$$

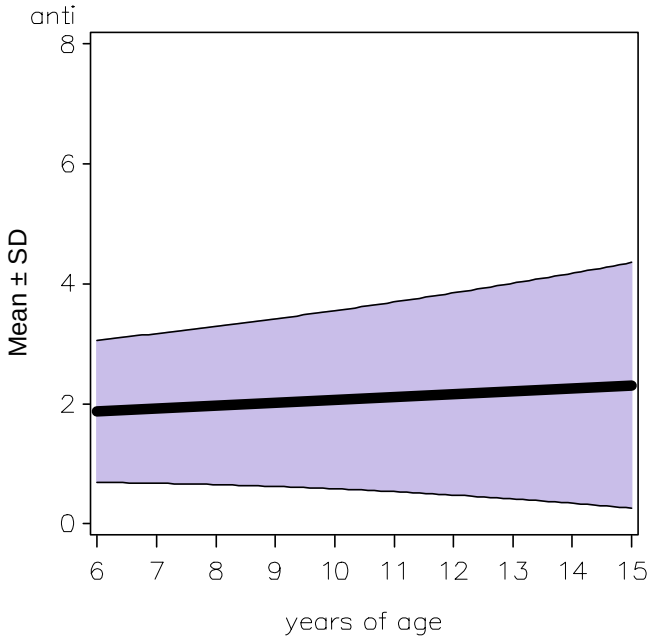
- Assumes continuous individual differences in change over time.

RCN Fit to Antisocial Data

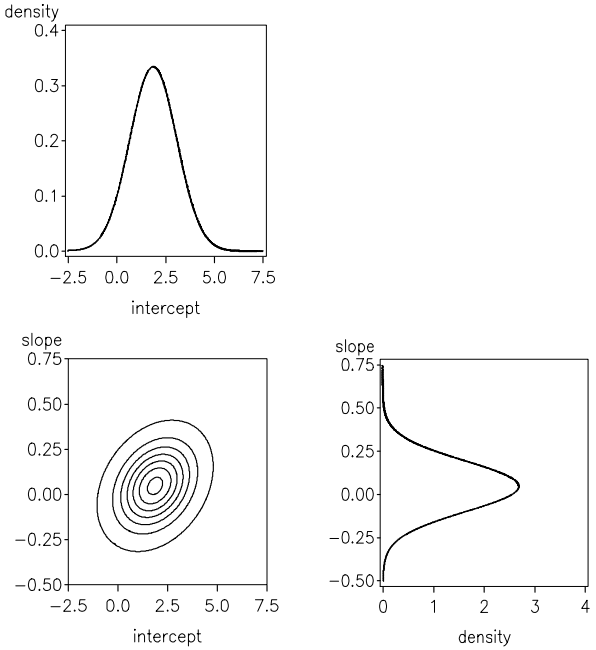
Parameter	Estimate (SE)
Fixed effects	
Intercept (γ_{00})	1.87 (0.07)**
Age (γ_{10})	0.05 (0.01)**
Variance / Covariance Parameters	
Intercept (τ_{00})	1.43 (0.25)**
Age (τ_{11})	0.02 (0.01)*
Covariance (τ_{10})	0.05 (0.04)
Residual (σ^2)	2.09 (0.14)**

Note. Robust standard errors reported.
* p < .05 ; ** p < .01

Graphical Depiction of RCN



Graphical Depiction of RCN



Adding Predictors to RCN

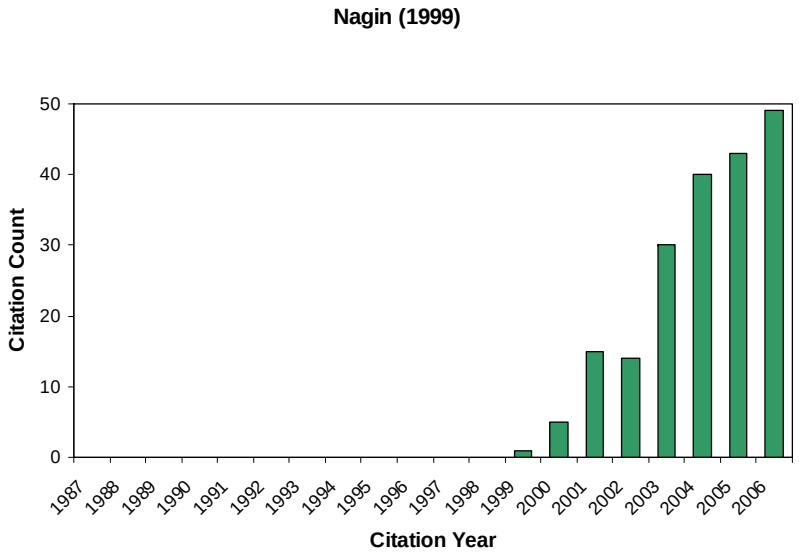
Parameter	Estimate (SE)
Prediction of Intercepts (β_{0i})	
Intercept	1.18 (0.14)**
Sex	**0.85** (0.13)**
Home	**-0.22** (0.04)**
Prediction of Slopes (β_{1i})	
Intercept	0.12 (0.03)**
Sex	0.01 (0.03)
Home	-0.01 (0.01)
Time Varying Covariates	
Math	**-0.17** (0.04)**
Variance / Covariance Parameters	
Intercept	1.05 (0.16)**
Age	0.02 (0.01)*
Covariance	0.06 (0.03)*
Residual	2.11 (0.06)**

* p<.05; ** p<.01

Three Types of RCGMs

1. RCN: Models that assume the random coefficients are normally distributed (conditional on predictors)
   - Latent curve/trajectory/growth models, HLM, multilevel growth models, mixed-effects models
2. RCD: Models that assume the random coefficients are discretely distributed across K groups
   - Semi-parametric group-based trajectory method, non-parametric random coefficient model, latent class growth analysis, latent class regression

Trends in Use of RCD Models (SPGBM)



RCD

- Within-Person Model:

$$AB_{ti} = \beta_{0i} + \beta_{1i}\left(Age_{ti} - 6\right) + r_{ti} \qquad r_{ti} \sim N\left(0, \sigma^2\right)$$

- Between-Person Model:

$$\beta_{0i} = \gamma_{00(k)} \text{ if } C_i = k \qquad C_i = 1, 2, \ldots, K$$
$$\beta_{1i} = \gamma_{10(k)} \text{ if } C_i = k \qquad P(C_i = k) = \pi_{(k)}$$

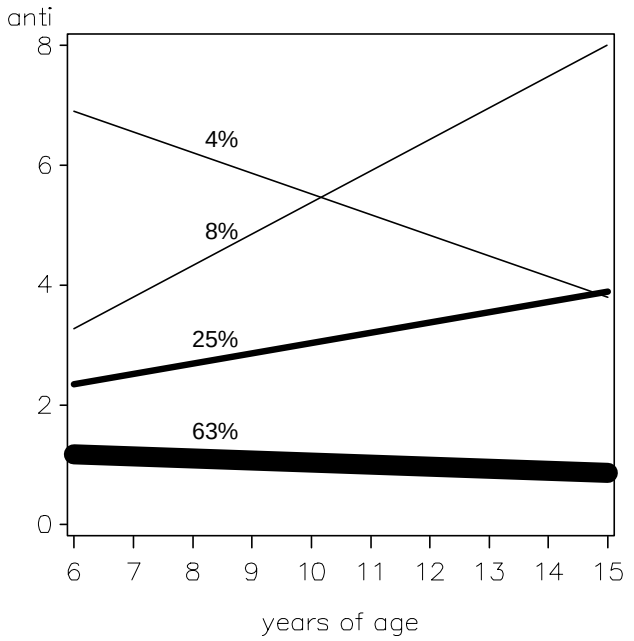
- Assumes individual differences in change over time are discretely distributed – K types of trajectories.

RCD Fit to Antisocial Data

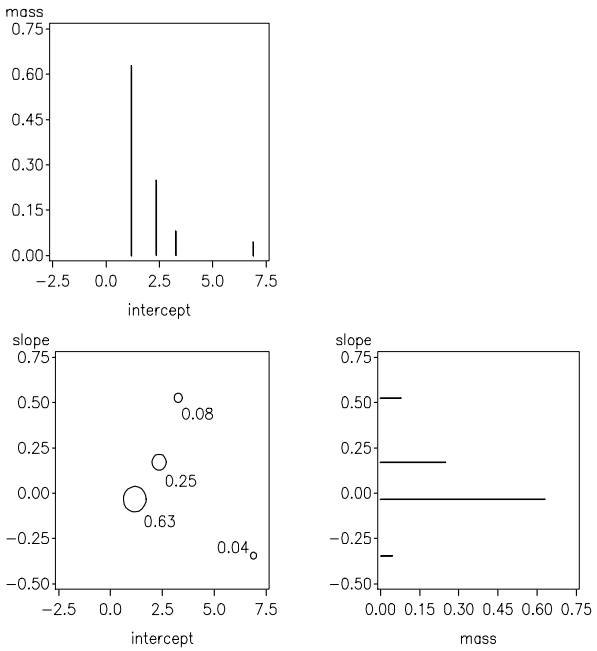
	Latent Class			
Parameter	"High Increasing"	"Low Increasing"	"High Declining"	"Abstaining"
Class Size				
Class probability ($\pi_{(k)}$)	.08	.25	.04	.63
Sample *N*	70	222	39	562
Fixed effects				
Intercept ($\gamma_{00(k)}$)	3.27 (0.91)**	2.35 (0.22)**	6.90 (1.95)**	1.17 (0.07)**
Age ($\gamma_{10(k)}$)	0.53 (0.20)**	0.17 (0.04)**	-0.34 (0.37)	-0.03 (0.01)*
Variance / Covariance Parameters				
Residual (σ^2)	2.02 (0.10)**	2.02 (0.10)**	2.02 (0.10)**	2.02 (0.10)**

Note. Robust standard errors reported.
* p < .05 ; ** p < .01

Graphical Depiction of RCD



Graphical Depiction of RCD



Adding Predictors to RCD

	Latent Class			
Parameter	"High Increasing"	"Low Increasing"	"High Declining"	"Abstaining"
Class Size[a]				
Class probability	.08	.28	.05	.59
Sample *N*	71	236	39	498
Between-Class Effects[b]				
Sex	**4.25** (2.15,8.43)	**3.70** (2.30,5.94)	**9.70** (3.15,29.88)	N/A
Home	**0.59** (0.47,0.72)	**0.65** (0.56,0.75)	**0.68** (0.48,0.96)	N/A
Within-Class Effects				
Intercept	3.08 (0.22)**	2.23 (0.14)**	6.77 (0.33)**	-0.91 (0.11)**
Age	0.56 (0.05)**	0.17 (0.03)**	-0.32 (0.07)*	0.03 (0.03)
Math	**-0.11** (0.04)**	**-0.11** (0.04)**	**-0.11** (0.04)**	**-0.11** (0.04)**
Variance / Covariance Parameters				
Residual	1.96 (0.04)**	1.96 (0.04)**	1.96 (0.04)**	1.96 (0.04)**

* p < .05 ; ** p < .01

[a] Based on estimated posterior probabilities

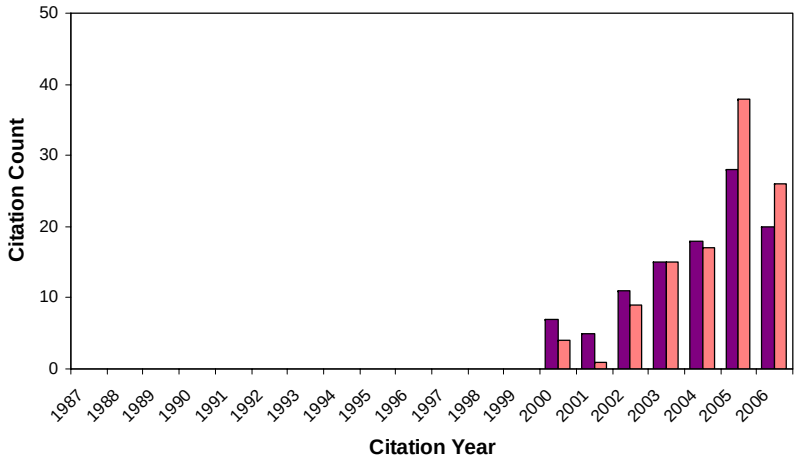
[b] Odds Ratio (95% CI); Reference class is Abstaining

Three Types of RCGMs

1. RCN: Models that assume the random coefficients are (conditionally) normally distributed
   - Latent curve/trajectory/growth models, HLM, multilevel growth models, mixed-effects models
2. RCD: Models that assume the random coefficients are discretely distributed across K latent groups
   - Semi-parametric group-based trajectory method, non-parametric random coefficient model, latent class growth analysis, latent class regression
3. RCNM: Models that assume the random coefficients are from a (conditional) normal mixture distribution with K latent groups
   - General growth mixture models

Trends in Use of RCNM Models (GMMs)

Muthen & Shedden (1999)
Muthen & Muthen (2000)



RCNM

- Within-Person Model:

$$AB_{ti} = \beta_{0i} + \beta_{1i}\left(Age_{ti} - 6\right) + r_{ti} \qquad r_{ti} \sim N\left(0, \sigma^2_{(k)}\right)$$

- Between-Person Model:

$$\beta_{0i} = \gamma_{00(k)} + u_{0i(k)} \text{ if } C_i = k$$
$$\beta_{1i} = \gamma_{10(k)} + u_{1i(k)} \text{ if } C_i = k$$

$$\begin{pmatrix} u_{0i(k)} \\ u_{1i(k)} \end{pmatrix} \sim N\left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \tau_{00(k)} & \\ \tau_{10(k)} & \tau_{11(k)} \end{pmatrix}\right]$$

$$C_i = 1, 2, \ldots, K$$
$$P(C_i = k) = \pi_{(k)}$$

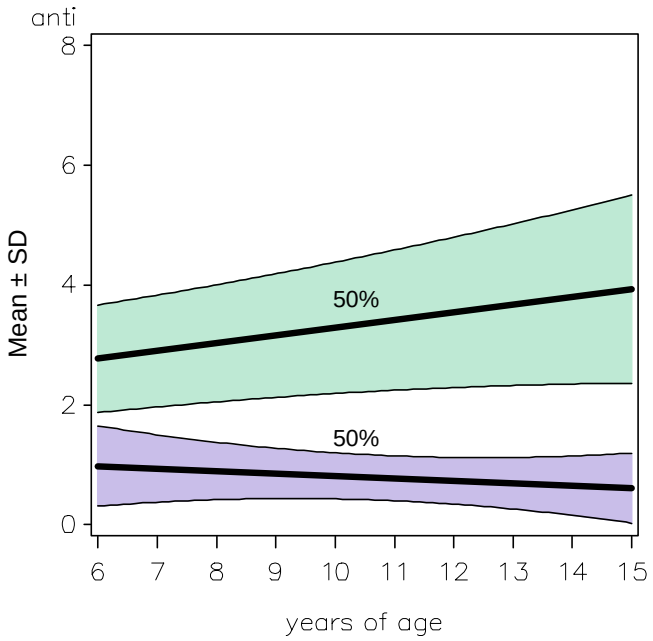
- Assumes K discrete trajectory groups, within which individual differences in change over time are continuously distributed.

RCNM Fit to Antisocial Data

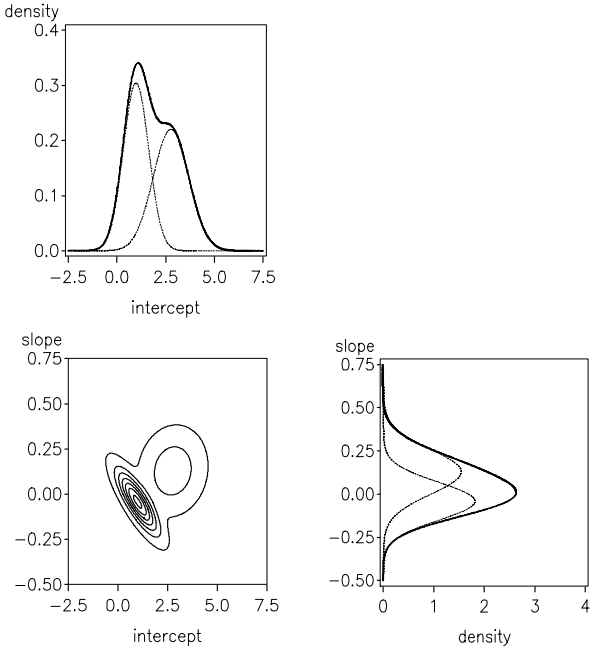
Parameter	Latent Class "Increasing"	Latent Class "Decreasing"
Class Size		
Class probability ($\pi_{(k)}$)	.50	.50
Sample N	445	449
Fixed effects		
Intercept ($\gamma_{00(k)}$)	2.78 (0.13)**	0.98 (0.09)**
Age ($\gamma_{10(k)}$)	0.13 (0.02)**	-0.04 (0.02)**
Variance / Covariance Parameters		
Intercept ($\tau_{00(k)}$)	0.83 (0.42)*	0.43 (0.10)**
Age ($\tau_{11(k)}$)	0.02 (0.02)	0.01 (0.004)*
Covariance ($\tau_{10(k)}$)	0.01 (0.07)	-0.06 (0.02)**
Residual ($\sigma^2_{(k)}$)	3.55 (0.32)**	0.62 (0.07)**

Note. Robust standard errors reported.
* p < .05 ; ** p < .01

Graphical Depiction of RCNM



Graphical Depiction of RCNM



Adding Predictors to RCNM

Parameter	Latent Class "Increasing"	Latent Class "Decreasing"
Class Size[a]		
Class probability	.51	.49
Sample *N*	426	417
Between-Class Effects[b]		
Sex	**2.82** (1.93,4.11)	N/A
Home	**0.66** (0.58,0.75)	N/A
Within-Class Prediction of Intercepts (β_{0i})		
Intercept	1.89 (0.24)**	0.76 (0.10)
Sex	**0.62** (0.24)**	**0.31** (0.12)**
Home	-0.06 (0.04)	-0.06 (0.04)
Within-Class Prediction of Slopes (β_{1i})		
Intercept	0.26 (0.05)**	0.01 (0.02)
Sex	-0.01 (0.05)	**-0.05** (0.03)*
Home	0.01 (0.01)	0.01 (0.01)
Time Varying Covariates		
Math	**-0.30** (0.07)**	-0.06 (0.03)
Variance / Covariance Parameters		
Intercept	0.45 (0.34)	0.35 (0.11)**
Age	0.01 (0.02)	0.01 (0.004)**
Covariance	0.05 (0.06)	-0.05 (0.02)**
Residual	3.60 (0.17)**	0.61 (0.05)**

* p < .05 ; ** p < .01
[a] Based on estimated posterior probabilities
[b] Odds Ratio (95% CI); Reference class is Decreasing

Comparison of Three Models

- Overall trends are similar
  - Most children had low levels of AB at age 6.
  - Some children showed increases in AB from age 6 to 15, others showed stable or decreasing AB.
  - Those with highest initial levels of AB tended to show greatest increases over time.
  - Male children, and children from less supportive home environments displayed higher levels of AB.
  - Children scoring poorly on the PIAT-M displayed higher levels of AB.
- Despite general consistency, much controversy over appropriate model

Controversy in the Selection of RCGMs

- 5 recent papers on RCD and RCNM models have been followed by commentaries and rejoinders
  - Bauer & Curran (2003, *Psychological Methods*), commentaries by Cudeck & Henly, Muthen, & Rindskopf.
  - Eggleston, Laub & Sampson (2004, *J. Quant. Criminology*), commentary by Nagin.
  - Nagin & Tremblay (2005, *Annals AAPSS*), commentaries by Maughan and Raudenbush.
  - Nagin & Tremblay (2005, *Criminology*), commentary by Sampson & Laub.
  - Connell & Frye (2006, *Infant & Child Dev*), commentaries by Hoeksma & Kelderman, Muthen, and Stanger.
- Reflects disagreements, misunderstandings about relative merits of different RCGMs.

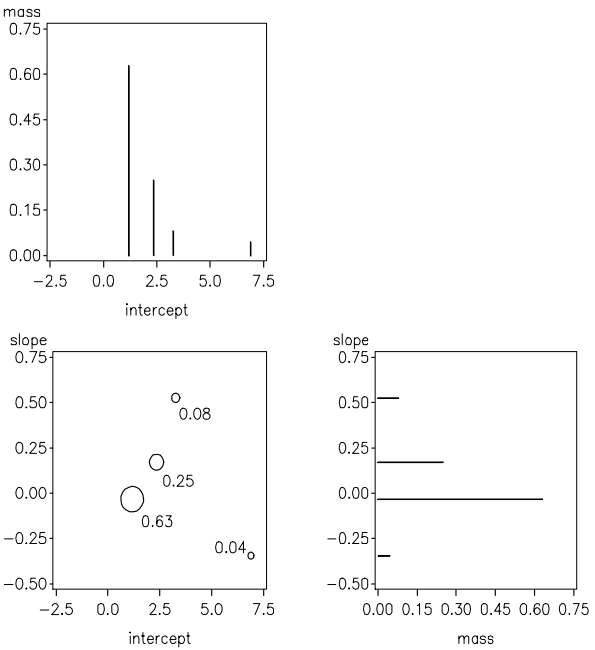
Key Issue: The Interpretation of Groups

- Often, the latent classes estimated from RCD or RCNM models are interpreted as true taxa (i.e., "real" groups).
- This can be problematic
  - Are there 4 groups (RCD), 2 groups (RCNM), or no groups (RCN)?
  - Should groups be strictly homogeneous (RCD) or do we permit within-group variability (RCNM)?
  - Spurious groups can compensate for errors in model specification, e.g., lack of normality of residuals (Bauer & Curran, 2003, 2004).
  - Number/nature of groups can change with minor alterations in model specification, covariates, measurement, or design (Eggleston et al, 2004; Jackson & Sher, 2005, 2006).
  - RCN does permit groups as a function of observed covariates (when strong etiological theory for taxa).

Key Issue: The Interpretation of Groups

- Nagin (2005) advocates use of groups as an approximating device even when groups are artificial.

Graphical Depiction of RCD



Key Issue: The Interpretation of Groups

- Nagin (2005) advocates use of groups as an approximating device even when groups are artificial.
- Drawbacks to Groups:
  - Artificial groups prone to reification.
  - Use of RCD and RCNM models may reduce power.
- Alternatives to Groups:
  - RCN models relatively robust to violation of normality assumption for random effects.
  - Other semiparametric RCGMs (Chen, Zhang & Davidian, 2002; Zhang & Davidian, 2001)
- Perhaps better to focus less on groups and more on overall trends.

Conclusions

- RCGMs offer many conceptual and statistical advantages for modeling individual change
- Many different RCGMs (HLM, LCM, GMM, LCGA, SPGBA, etc) can be organized into three categories
  - RCN assumes continuously distributed individual differences.
  - RCD assumes discretely distributed individual differences.
  - RCNM assumes continuously distributed individual differences within a small number of discrete groups.
- Key issue in choosing between models is utility of groups
  - Groups correspond nicely to taxonomic theory
  - Groups often improve model flexibility and fit.
  - Interpretation of latent groups can be risky.