Predictions of Individual Change Recovered With Latent Class or Random Coefficient Growth Models

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Popular longitudinal models allow for prediction of growth trajectories in alternative ways. In latent class growth models (LCGMs), person-level covariates predict membership in discrete latent classes that each holistically define an entire trajectory of change (e.g., a high-stable class vs. late-onset class vs. moderate-desisting class). In random coefficient growth models (RCGMs, also known as latent curve models), however, person-level covariates separately predict continuously distributed latent growth factors (e.g., an intercept vs. slope factor). This article first explains how complex and nonlinear interactions between predictors and time are recovered in different ways via LCGM versus RCGM specifications. Then a simulation comparison illustrates that, aside from some modest efficiency differences, such predictor relationships can be recovered approximately equally well by either model—regardless of which model generated the data. Our results also provide an empirical rationale for integrating findings about prediction of individual change across LCGMs and RCGMs in practice.

Keywords: group-based trajectory model, interaction, latent curve model, latent class growth model, person-oriented methods, prediction, random coefficient growth model

Two popular methods for modeling change in the social sciences are latent curve models—also called random coefficient growth models (RCGMs; e.g., Bollen & Curran, 2006; Goldstein, 2003; Raudenbush & Bryk, 2002) and latent class growth models (LCGMs; also called semiparametric group-based trajectory models; e.g., Muthén, 2001; Nagin, 1999). These models are most commonly distinguished based on how they describe interindividual differences in intraindividual change. LCGMs allow variation in developmental change across discrete, homogenous person-groups (latent classes), whereas RCGMs allow variation in developmental change across continuously distributed growth factors.

These models can also be distinguished based on how they use observed characteristics of the individual to predict his or her growth trajectory. In RCGMs, person-level covariates can predict one or more continuously distributed growth factors. Prediction of the intercept factor entails a main effect; prediction of slope factors entails implicit interactions with time (Curran, Bauer, & Willoughby, 2004). On the other hand, in LCGMs, person-level covariates predict a discretely distributed latent classification variable. Effects of these covariates differ across categories of the latent classification variable to implicitly accommodate a variety of predictive relations of unspecified functional form.

This article compares how well LCGM and RCGM methods can recover complex, potentially nonlinear interactions between person-level characteristics and time. Recovery of such interactions is often a central aim of developmental research—particularly in person-oriented (e.g., Bergman & Trost, 2006; Cairns, Bergman, & Kagan, 1998; Sterba & Bauer, 2010a, 2010b) and interactional (Magnusson, 1985, 1990) research paradigms. One line of thinking is that LCGMs should have an inherent advantage for recovering such effects. More generally, classification-based methods (that extract discrete classes or groups of people, such as LCGMs) have historically been thought inherently advantageous for recovering nonlinear or highly interactive predictive relationships, compared to their non-classification-based counterparts (such as RCGMs; Bergman, 2001;
Bergman & Magnusson, 1997; Bergman & Trost, 2006; Connell, Dishion, & Deater-Deckard, 2006; Laursen & Hoff, 2006; Moffitt, 2006, 2008; Muthén, 2001, 2004; Nagin & Tremblay, 2005b; Pastor, Barron, Miller, & Davis, 2007; Segawa et al., 2005). Anticipated variations of classification-based methods such as LCGMs have been attributed to the perspective that models like RCGMs can only accommodate linear predictive relationships (e.g., Hill, White, Chung, Hawkins, & Catalano, 2000; Krišman, 2003; Shaw & Liang, 2012; Torppa, Poikkeus, Laakso, Ekblad, & Lyttinen, 2006) or attributed to the greater flexibility of classification methods such as LCGMs in accounting for complex predictor relationships (Muthén, 2004; Pastor et al., 2007). For instance, regarding flexibility differences, variants of RCGMs are considered to have more difficulty recovering a “nonmonotonic intervention effect that exists for children of medium-range aggression and is absent for the most or least aggressive children” and more generally when “the effect of a covariate is not strong or even present except in a limited range of the growth factor or outcome” (Muthén, 2004, p. 353).

Alternatively, observed differences in predictive relationship recovery across the two models might have more to do with conventions for how the models are applied in practice, than with inherent capabilities of the models. Restrictive specifications of RCGMs are common that, for instance, do not include nonlinear or higher order interactions (e.g., Chen & Cohen, 2006; Farrell & Sullivan, 2004; Hirsh-Pasek & Burchinal, 2006; Hox, 2007; Kerr & Michalski, 2007; Peugh, 2010; Siller & Sigman, 2008; Singer, 1998; van Oort, Greaves-Lord, Verhulst, Ormel, & Huizink, 2009). These specifications thus preclude the possibility of accommodating complex, nonlinear interactions. On the other hand, very unrestricted and flexible specifications of LCGMs are common, with all predictors having effects varying across all classes (e.g., Dush, Taylor, & Kroeger, 2008; Gross, Shaw, Burwell, & Nagin, 2009; Lacourse et al., 2006; Muthén & Muthén, 2000; Nagin, 2005; Paciello, Fida, Tramontano, Lupinetti, & Caprara, 2008; Pickles & Croudace, 2010; Wiesner & Kim, 2006; Zhang, Mitchell, Bambauer, Jones, & Prigerson, 2008). This specification implicitly accommodates a wide variety of complex nonlinear interactions. Nonetheless, methodology exists for incorporating nonlinear and/or interactive predictor effects in RCGMs (Aiken & West, 1991; Curran et al., 2004, 2006). Although these strategies are still routinely viewed as limited (e.g., Bergman, 2001; Bergman & El Khouri, 2003; Laursen & Hoff, 2006; Magnusson, 1998; Pastor et al., 2007), their capability for recovering complex, potentially nonlinear interactions in RCGMs has not been subject to careful empirical comparison with LCGMs.

Two concerns motivate our comparison of the ability of LCGMs and RCGMs to recover the effects of predictors on individual trajectories. First, researchers might want to know how well they can predict how a person’s behavior changes over time when the unobserved discrete or continuous nature of individual differences in growth is potentially misspecified (e.g., Butler & Louis, 1992; Cudeck & Henly, 2003; Raudenbush, 2005). This misspecification possibility is salient because existing selection criteria and diagnostics for empirically discriminating between continuous and discrete individual differences have been shown to effectively do so under relatively narrow conditions without robustness to various real-world data characteristics (Bauer, 2007; Bauer & Curran, 2003a, 2003b, 2004; Lubke & Neale, 2006, 2008). Furthermore, substantive theory might be nonspecific or incorrect about whether individual differences are discrete (trajectory classes) or continuous (growth factors). Second, previous comparisons involving models similar to LCGMs and RCGMs have been rare and have suffered from two limitations. Comparisons have been impeded by a lack of common criteria for model evaluation and have involved conditions that unduly favor one method. To resolve uncertainties about the relative ability of LCGMs and RCGMs for predictive relationship recovery in a manner that overcomes these limitations, this article addresses three questions:

1. What metric can be used for comparing predictive relationships, and their recovery, across LCGMs and RCGMs?
2. Can the same predictive relationships be recovered regardless of the match between fitted model (LCGM or RCGM) and generating model (LCGM or RCGM)?
3. Are there particular data conditions under which only one model (LCGM or RCGM) can accurately recover predictive relationships?

The first question is addressed following a brief review of the LCGM and RCGM. The second question is addressed in Study 1, and the third in Study 2.

MODEL SPECIFICATIONS

Random Coefficient Growth Model

A conditional RCGM is given as

\[ y_i = X_i \beta_i + \varepsilon_i \]  \hspace{1cm} (1)

\[ \beta_i = W_i \gamma + u_i \] \hspace{1cm} (2)

Considering first Equation 1, \( y_i \) is a \( T \times 1 \) vector of person \( i \)’s repeated measures for time points \( t = 1 \ldots T \); \( X_i \) is a \( T \times p \) matrix of Level 1 predictors (here, including a vector of \( 1 \)s, vector of time scores, \( t_i \) \ldots \( t_{iT_i} \), and potentially, vectors of powered time scores); \(^1\) and \( \beta_i \) is a \( p \times 1 \) vector of growth factors interpretable as intercept, linear,

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\(^1\) Adding time-varying predictors is also possible but is not the focus of this article.
and higher order slope growth coefficients for each person. 

\[ \mathbf{e}_i \] denotes a \( T \times 1 \) vector of time-specific residuals where 

\[ \mathbf{e}_i \sim N(\mathbf{0}, \Sigma) \]

and usually \( \Sigma = \sigma^2 \mathbf{I} \). The growth coefficients in \( \mathbf{b}_i \) can each then be regressed on person-level predictors, as shown in Equation 2. \( \mathbf{W}_i \) is a \( p \times p(1 + m) \) block diagonal matrix containing Level 2 (person-level) predictors in each block. \( \mathbf{y} \) denotes a \( p(1 + m) \times 1 \) vector of fixed effects (i.e., conditional means of growth coefficients and effects of person-level predictors on growth). \( \mathbf{u}_i \) is a \( p \times 1 \) vector of random effects\(^2\) (e.g., individual differences in growth) where \( \mathbf{u}_i \sim N(\mathbf{0}, \mathbf{T}) \) and \( \mathbf{T} \) is an unstructured \( p \times p \) covariance matrix of random effects. Substituting Equation 2 into Equation 1 yields the reduced form:

\[ \mathbf{y}_i = \mathbf{X}_i \mathbf{W}_i \mathbf{y} + \mathbf{X}_i \mathbf{u}_i + \mathbf{e}_i \]  

(3)

The multiplication of \( \mathbf{W}_i \) with \( \mathbf{X} \) introduces potential interaction effects between person-level predictors and functions of time. For example, consider a special case of the reduced form model, expressed for person \( i \) at time \( t \), where we have one person-level predictor \( w_i \) and linear growth:

\[ y_{it} = (\gamma_{00} + \gamma_{01} w_i + u_{0i}) + (\gamma_{10} + \gamma_{11} w_i + u_{1i}) t + e_{it} \]  

(4)

This expression clarifies that \( w_i \) affecting the intercept coefficient constitutes a main effect, whereas \( w_i \) affecting a slope coefficient constitutes a cross-level product interaction (here \( \gamma_{11} w_i t \)).

Latent Class Growth Model

A conditional LCGM for person \( i \) in class \( k \) is given as

\[ \mathbf{y}_{i|c_i=k} = \mathbf{X}_i \mathbf{y}^{(k)} + \mathbf{e}_i \]  

(5)

\[ \pi_{ik}(\mathbf{w}_i) = \frac{\exp(\delta_{ik}^{(k)} + \mathbf{w}_i) \Sigma_{ik}}{\sum_{k=1}^{K} \exp(\delta_{ik}^{(k)} + \mathbf{w}_i) \Sigma_{ik}} \]  

(6)

In Equation 5, \( c_i \) is a categorical latent variable that can take on values \( k = 1 \ldots K \). \( \mathbf{e}_i \) is again a \( T \times 1 \) vector of time-specific residuals where \( \mathbf{e}_i \sim N(\mathbf{0}, \Sigma) \) and \( \Sigma = \sigma^2 \mathbf{I} \). \( \mathbf{y}^{(k)} \) is a \( p \times 1 \) vector of class-specific growth coefficient values. If, for instance, intercept, linear, and quadratic aspects of change are desired for Class 1 but only intercept and linear aspects for Class 2, the third element of \( \mathbf{y}^{(k)} \) could be fixed to 0 in Class 2. In Equation 6, the proportion of individuals in class \( k \), \( \pi_{ik}(\mathbf{w}_i) \), is modeled as a function of covariates using a multinomial logistic regression. Here, \( \delta_{0k}^{(k)} \) is a scalar multinomial intercept; \( \mathbf{w}_i \) is an \( m \times 1 \) vector of person-level predictors of class membership; and \( \delta_{ik}^{(k)} \) is a \( 1 \times m \) vector of multinomial slopes capturing effects of these person-level predictors on class membership.

A reduced form LCGM is given as:

\[ \mathbf{y}_i = \sum_{k=1}^{K} c_{ik}(\mathbf{X}_i \mathbf{y}^{(k)} + \mathbf{e}_i) \]  

(7)

\( c_{ik} \) is a class indicator label\(^3\) that can take on values of 0 or 1 (\( c_{ik} = 1 \) if a member of class \( k \); else \( c_{ik} = 0 \)). The expected value of the class indicator label is the proportion of persons in class \( k \), \( E(c_{ik}|\mathbf{w}_i) = \Pr(c_{ik} = 1|\mathbf{w}_i) = \pi_{ik}(\mathbf{w}_i) \).

Comparison of Model Specifications

Person-level covariates separately predict continuous variability in each aspect of change in the RCGM (Equation 2). Residual associations between aspects of change are captured within \( \mathbf{T} \). Consequently, this model readily provides specific details on which main or interaction effects of predictors are statistically significant and more versus less important. However, the onus is on the researcher to reintegrate this information to obtain a holistic understanding of predictive relations (Magnusson, 1998). On the other hand, person-level covariates predict the entire trajectory as a whole in the LCGM (Equation 6). Consequently, this model readily provides a coherent overall depiction of predictive relationships—but at the expense of indicating whether these relationships entail main effects or interaction effects with time.

Comparison of Equation 3 with Equations 5 and 6 indicates that to accommodate more complex (higher order, nonlinear) person-level predictor relations, RCGM and LCGM need different things. Generally, the LCGM needs more classes, higher order growth coefficients within class, and more class-varying growth and multinomial coefficients; allowing differing predictor effects across class-varying time trends indirectly captures complex interactions of predictors and time. On the other hand, the RCGM needs higher order product and power terms (e.g., \( w_i t \) \( \pi_{01}^{(k)} \), \( w_i^2 \) \( \pi_{11}^{(k)} \)).

Furthermore, Equations 6 and 7 show that person-level covariates enter the model nonlinearly in the LCGM, in contrast to the RCGM. For any given data set, the functional form for the predictor effects must be misspecified for at least one of the two models. In other words, fitting an RCGM when LCGM is the true model (or vice versa) not only represents a misspecification of distributional form (the discrete or continuous nature of individual differences) but also implies a different functional form (i.e., the nature of relationships of predictors to the growth trajectories). Consequently, in

\(^2\)In this notation, all Level 1 time scores have random effects; setting elements of \( \mathbf{u}_i \) to 0 would allow some time scores to only have fixed effects.

\(^3\)A vector consisting of all \( K \) class indicator labels \( c_{1i} \ldots c_{Ki} \) conforms to a multinomial distribution consisting of 1 draw on \( K \) categories with probabilities \( \pi_{1i}(w_i) \ldots \pi_{Ki}(w_i) \) (McLachlan & Peel, 2000).
theory all moments could be misspecified. This point is important because if only the distributional form of individual trajectories is misspecified, the fixed effect and variance component estimates of the model are consistent, when repeated measures are conditionally normally distributed (Verbeke & Lessafre, 1997). Circumstances minimizing the extent and consequences of functional form misspecification are discussed and investigated later.

WHAT METRIC CAN BE USED FOR COMPARING PREDICTIVE RELATIONSHIPS, AND THEIR RECOVERY, ACROSS LCGMS AND RCGMS?

Prior studies comparing recovery of predictor effects using longitudinal classification-based versus non-classification methods (that are more restrictive relatives of RCGMs; e.g., repeated measures analysis of variance) have had key limitations. Sometimes empirical data comparisons took place without including key higher order effects in the nonclassification model (e.g., von Eye & Bogat, 2006; see also Magnusson, 1998). Because classification-type models like LCGMs do not require the inclusion of explicit power or product terms to account for certain nonlinearities or interactions, but models like RCGMs do, such comparisons are weighted in favor of the former. Other times, predictive relationship recovery was not assessed on a comparable metric across fitted models (Bogat, 2009; von Eye & Bogat, 2006; von Eye, Bogat, & Rhodes, 2006).

An inclusive, non-model-specific metric is needed to compare RCGM and LCGM performance. An obstacle is that the coefficients estimated to capture the effects of person-level predictors lack a one-to-one correspondence across the models. Calculating and comparing predicted trajectories from each model overcomes this obstacle. Predicted trajectories are in the scale of the repeated measure. Thus, they are directly comparable even though the individual parameter estimates from which they were computed are not. Previously, predicted profiles have been calculated for related cross-sectional models in a single-sample setting (Bauer & Shanahan, 2007). In a longitudinal context, predicted trajectories have been calculated for one model or the other, not both.

Predicted Trajectories for RCGMs

In the RCGM literature, it is increasingly common to depict predictive relationships by plotting predicted trajectories obtained as the expected value of Equation 3 at chosen values of person-level predictor(s) (Curran et al., 2004; 2006):

\[
E(y_i | W_i) = X_i W_i \gamma
\]

This follows the logic of plotting predicted regression lines to aid in visualizing complex interaction effects in multiple regression (Aiken & West, 1991).

Predicted Trajectories for LCGMs

In the LCGM literature, in contrast, predictive relationships are commonly interpreted by exponentiating multinomial slopes in Equation 6 to indicate the multiplicative change in the odds of belonging to class \( k \) versus the reference class per a one unit increment in the predictor. Recently, Nagin and Tremblay (2005a, pp. 883, 885) stated “grave reservations” about this practice because “even if the groups are thought of as real entities, it is not possible to assign individuals definitively to a specific trajectory ex ante based on number of risk factors. It is possible to construct only an expected trajectory.” Hence, they suggested computing predicted trajectories by taking the expected value of Equation 7, at particular predictor values:

\[
E(y_i | W_i) = \sum_{k=1}^{K} \pi_i^{(k)}(W_i)X_i \gamma^{(k)}
\]

These predicted trajectories can be plotted to better understand how predictors influence individual change over time. Nagin and Tremblay (2005a) and Bauer and Shanahan (2007) emphasized that such predictions are interpretable whether discrete classes are literally thought to exist, or are construed only as an approximation. However, this procedure is currently underused in LCGM applications.

STUDY 1: CAN THE SAME PREDICTIVE RELATIONSHIPS BE RECOVERED REGARDLESS OF THE MATCH BETWEEN FITTED MODEL (LCGM OR RCGM) AND GENERATING MODEL (LCGM OR RCGM)?

The previous section proposed using predicted trajectories to overcome one limitation of prior research: lack of an inclusive, non-model-specific metric for comparing predictive relationship recovery across LCGMs and RCGMs. Another limitation of prior research is that the models’ performance has not been evaluated with respect to a gold standard that is (a) known, and (b) equally difficult for both models (e.g., von Eye & Bogat, 2006). Regarding (a), prior comparisons of predictive relationship recovery across models related to LCGMs and RCGMs have used empirical data. Simulating data from a population model provides the opportunity for
a known gold standard against which to compare predicted relationship recovery for each fitted model. Regarding (b), however, using simulated data raises the issue of which model to choose as a generating model. If certain kinds of predictor relationships are more easily recovered by one model than the other, choosing LCGM as the generating model could disadvantage RCGM—and vice versa. To overcome this issue, in Study 1 we used both LCGM and RCGM as generating models, and related these generating models in a special way so that (a) indices such as bias and mean squared error are on the same metric regardless of generating model, and (b) model misspecification is equated regardless of whether LCGM is fit to data generated from RCGM (or RCGM is fit to data generated from LCGM). This special relatedness, here termed reversibility, is accomplished in the following manner. The fitted RCGM, when classes exist, becomes the generating model, when continua exist. Also, the fitted LCGM, when continua exist, becomes the generating model, when classes exist. Details are provided later.

In sum, we overcome prior limitations by (a) comparing predicted relationship recovery using an inclusive, non-model-specific metric (predicted trajectories), and (b) employing an equally difficult, known gold standard (“reversibility” conditions). Study 1 uses this approach to investigate relative accuracy and efficiency of LCGM and RCGM predictive relationship recovery.

Hypotheses

As mentioned previously, the LCGM and RCGM imply different functional forms for the effects of predictors and thus they cannot both represent these effects with perfect accuracy. In a simulation, the true model then should have some advantage regarding bias because it contains no approximation error. But, practically speaking, with sufficiently many classes and class-varying coefficients, Equation 9 could approximate Equation 8 with little bias, even if RCGM is generating. Similarly, with sufficiently many power and product terms included, Equation 8 could approximate Equation 9 with little bias, even if LCGM is generating.

Hypothesis 1. Predicted trajectories should be recoverable with little bias, regardless of which of the two models actually generated the data.

Irrespective of bias, the more parsimonious model (e.g., that with fewer estimated parameters) should have an advantage with respect to sampling variability. The RCGM specifications used in our simulations, which include higher order terms, have as many or more parameters than those commonly encountered in practice (e.g., Hox, 2007). In contrast, LCGM specifications used in our simulations are parameterized like those commonly encountered in practice (e.g., Nagin, 2005). Nevertheless, these RCGMs still had fewer parameters than the corresponding LCGMs (see Table 1). To balance potential differences in both sampling variability and bias, we examine the mean squared error (MSE). When neither the true nor misspecified model results in large bias, sampling variability can dominate MSE. This favors the more efficient model, particularly at smaller N (Cattin, 1978; Einhorn & Hogarth, 1975).

Hypothesis 2. (a) For RCGM-generated data, predicted trajectory estimates from LCGM should have higher MSE than corresponding estimates from RCGM. (b) For LCGM-generated data, predicted trajectory estimates from RCGM need not exceed the MSE from LCGM estimates, depending on N.

Hypothesis 2 implies that, if there is a risk of model misspecification, misspecifying continua as classes may have some greater MSE cost than misspecifying classes as continua.

Methods

The Study 1 simulation has 24 cells from four crossed conditions: generating model (LCGM or RCGM), fitted model (LCGM or RCGM), number of predictors (1-predictor or 2-interacting predictors), and N (250, 500, 1,000). The range of Ns is representative of most LCGM applications (Sterba,

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<td><strong>Generating Model</strong></td>
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<td>LCGM one-predictor</td>
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<td>LCGM two interacting predictors</td>
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Note. LCGM = latent class growth model; RCGM = random coefficient growth model.
Baldasaro, & Bauer, 2012). Five hundred samples per cell were generated in SAS 9.2. Models were fit in Mplus 5.2 with maximum likelihood—using 200 sets of random starting values for LCGM.

**Population Generating and Fitted Models**

**One-predictor LCGM.** The one-predictor LCGM,

\[
y_{it}|c_{i}=k = \gamma_{00} + \gamma_{10} t_{it} + \gamma_{20} t_{it}^2 + \varepsilon_{it} \tag{10}
\]

where \( \varepsilon_{it} \sim N(0, \sigma^2) \), had generating parameters based on the aggressive behavior empirical application from Nagin and Tremblay (2005a), in which \( T = 7 \) and \( K = 4 \) (shown in Figure 1). An environmental risk variable \( r_{i} \sim BIN(8, .36) \) predicted class membership. See Online Appendix Table A.1 for generating parameters and implied conditional class probabilities. Among LCGM applications, similar patterns of unconditional trajectories are found in Cote, Tremblay, Nagin, Zoccolillo, and Vitaro (2002); Dodge, Du, Saxton, and Ganguli (2006); Laub, Nagin, and Sampson (1998); Séguin et al. (2007); and Shaw, Lacourse, and Nagin (2005). Similar cumulative risk predictor processes are seen in Jones and Nagin (2007) and Nagin (2005). Data generated from this LCGM were fit with the true four-class LCGM in Equation 10 as well as the following RCGM:

\[
y_{it} = \gamma_{00} + \gamma_{01} r_{i} + u_{0i} + (\gamma_{10} + \gamma_{11} r_{i} + u_{1i}) t_{it} + (\gamma_{20} + \gamma_{21} r_{i}) t_{it}^2 + \varepsilon_{it} \tag{11}
\]

where \( \varepsilon_{it} \sim N(0, \sigma^2) \) and \( u_{i} \sim N(0, T) \). Relevant product terms were included (cross-level interactions of risk with \( t_{it} \) and \( \varepsilon_{it}^2 \)). The quadratic coefficient was fixed because its random effect variance was not estimable.

**One-predictor RCGM.** To fulfill the reversibility criteria mentioned earlier, the generating parameters for the one-predictor RCGM were taken from average estimates obtained from fitting Equation 11 to the LCGM data. These parameters are given in Online Appendix Table A.2. In turn, the data generated from the one-predictor RCGM were fit with the generating RCGM in Equation 11 and with the LCGM in Equation 10, with various numbers of classes.

**Two-interacting-predictors LCGM.** The generating one-predictor LCGM was expanded to include not only an environmental risk, \( r_{i} \), predicting class membership, but also a binary genetic marker, \( g_{i} \sim BIN(1, .25) \), which interacts with \( r_{i} \), to constitute a gene \( \times \) environment interaction.

\[
y_{it}|c_{i}=k = \gamma_{00} + \gamma_{10} t_{it} + \gamma_{20} t_{it}^2 + \varepsilon_{it} \tag{12}
\]

\[
\pi_{i}^{(k)}(r_{i}, g_{i}) = \frac{\exp(\delta_{0}^{(k)} + \delta_{1}^{(k)} r_{i} + \delta_{2}^{(k)} g_{i} + \delta_{3}^{(k)} r_{i} g_{i})}{\sum_{k=1}^{K} \exp(\delta_{0}^{(k)} + \delta_{1}^{(k)} r_{i} + \delta_{2}^{(k)} g_{i} + \delta_{3}^{(k)} r_{i} g_{i})}
\]

Here, \( \varepsilon_{it} \sim N(0, \sigma^2) \). Specifically, when the protective allele is present, \( g_{i} = 1 \), it reduces the negative effect of environmental risk on aggression, such as described by Caspi et al. (2002). When \( g_{i} = 0 \), the protective allele is absent, as depicted in the conditional class probabilities and generating parameters in Online Appendix Table A.3. Recovery of such interactive effects is highly important to social science researchers. Data generated from this two-interacting-predictor LCGM were fit with the true four-class LCGM (Equation 12) and the RCGM in Equation 13:

\[
y_{it} = \gamma_{00} + \gamma_{01} r_{i} + \gamma_{02} g_{i} + \gamma_{03} r_{i} g_{i} + u_{0i} + (\gamma_{10} + \gamma_{11} r_{i} + \gamma_{13} r_{i} g_{i} + u_{1i}) t_{it} + \gamma_{20} + \gamma_{21} r_{i} + \gamma_{23} r_{i} g_{i}) t_{it}^2 + \varepsilon_{it} \tag{13}
\]

where \( \varepsilon_{it} \sim N(0, \sigma^2) \) and \( u_{i} \sim N(0, T) \). Relevant product terms were included (cross-level interactions of risk, and gene, and \( r_{i} g_{i} \) with \( t_{it} \) and \( t_{it}^2 \)). Again, the quadratic coefficient was fixed because its random effect variance was not estimable.

FIGURE 1 Study 1 unconditional latent class growth model trajectories (modeled after Nagin & Tremblay, 2005a). Note. Class 1 = Low; Class 2 = Moderate-declining; Class 3 = High-declining; Class 4 = Chronic. Figure 1 is based on our generated data, and is not a reproduction from the original empirical application.
Two-interacting-predictors RCGM. To fulfill the reversibility criteria mentioned earlier, the generating parameters for the two-interacting-predictor RCGM were average estimates obtained from fitting Equation 13 to the LCGM data. These parameters are given in Online Appendix Table A.4. In turn, the data generated from the two-interacting-predictors RCGM were fit with the true RCGM (Equation 13) and the LCGM in Equation 12, with various numbers of classes.

Data Analysis

Predicted trajectories per sample were calculated by substituting estimates into Equation 8 or Equation 9, depending on whether RCGM or LCGM was fit. Bias in predicted trajectories is depicted graphically by overlaying true conditional trajectories versus those averaged across samples. True conditional trajectories were obtained by substituting population parameters into either Equation 8 or Equation 9, depending on whether RCGM or LCGM was generating. MSE is tabled for each cell as

\[
MSE = \frac{1}{500} \times \frac{1}{N} \times \frac{1}{7} \\
\times \sum_{s=1}^{500} \sum_{i=1}^{N} \sum_{t=1}^{7} \left( E(y_{sit}|w_{is}, x_{sit}; \hat{\theta}) - E(y_{sit}|w_{is}, x_{sit}; \theta_{true}) \right)^2
\]

(14)

where \( s \) denotes sample (replication); \( N \) denotes sample size; \( \theta_{true} \) and \( \hat{\theta} \) denote generic vectors of generating and estimated parameter values, respectively, for sample \( s \); and \( w_{is} \) generically denotes the values of the set of predictor(s) included. The special reversibility relationship between the generating RCGM and LCGM gave their dependent variables approximately the same marginal variance.

Thus, MSEs can be directly compared across or within generating model. Standardized root MSEs, \( \sqrt{MSE/SD(y)} \), were also calculated to allow MSE differences to be interpretable in units of standard deviation of the outcome variable.

Results

Convergence and class selection. Convergence was \( \geq 97.8\% \). Regarding class selection, when LCGM was generating, the fitted LCGM had the true \( K \) (i.e., \( K = 4 \)), but when RCGM was generating, the fitted LCGM had the best Bayesian Information Criteria (BIC) \( K \) for that sample, shown in Table 2.

Bias. Because bias plots showed a similar pattern regardless of \( N \), only plots for \( N = 500 \) are provided (plots for other \( N \)s are available in the Online Appendix). In each plot in Figures 2 and 3, dashed lines are true and solid lines are average model-implied predicted trajectories. The lowest pair of dashed and solid predicted trajectories is for risks = 1; higher pairs of predicted trajectories are for risks = 3, 5, and 7, respectively. The disparity between the dashed and solid trajectories in each pair indicates bias. Figure 2 contains four plots, corresponding with the four combinations of generating and fitted one-predictor models. Overall, for one-predictor models, predicted trajectories are recovered with little bias regardless of generating or fitted model, in

An alternative strategy to using the per-sample best fitting number of classes would be to use the across-samples average best fitting number of classes in these calculations. Both strategies yielded the same overall pattern of findings regarding hypotheses.
line with Hypothesis 1. Some small bias is only found for high (rare) values of the risk predictor when a misspecified model is fit. This bias for extreme risk values is slightly worse when misspecifying continuous individual differences as classes than when misspecifying classes as continua. Still, such small bias is unlikely to alter a researcher’s overall conclusions, as the general pattern of predicted trajectories is always recovered.

In Figure 3, each row of plots corresponds with a design cell for the two-interacting-predictor models. Now predicted trajectories are not only provided by number of risks, but also by genetic status (left column = protective allele absent; right column = protective allele present, which suppresses aggression). Figure 3 results are largely similar to those in Figure 2. That is, regardless of generating or fitted model, little bias occurs, in line with Hypothesis 1. The little amount of bias again is observed for extreme (uncommon) predictor configurations and arises more often when misspecifying continuous individual differences as classes than classes as continua (second row). In contrast to the one-predictor case, some bias is also observed for extreme predictor values when fitting a correctly specified LCGM (first row).

MSE. Table 3 depicts MSE for predicted trajectories in all cells, using Equation 14. Within a given generating model, MSE was not always better when fitting a correctly specified model, compared to a misspecified model. Specifically, when RCGM was the generating model, fitting a misspecified LCGM resulted in worse MSE at all N, in line with Hypothesis 2a. In contrast, when LCGM was the generating model, fitting a misspecified RCGM resulted in 4% to 14% better MSE than a correct LCGM at low N, but resulted in worse MSE at high N, in line with Hypothesis 2b. At low N, the better MSE when fitting a misspecified RCGM is due to its relatively smaller sampling variability (particularly for extreme predictor values, as depicted in Figure 4) overshadowing its slightly larger bias. As N increases, sampling variability for both fitted models decreases, but bias differences remain, so MSE increasingly favors the correct model. Still, from a practical perspective, both models provided good fit. Also, MSE differences across fitted models were small (≤.01 SD in y, in the far right column of Table 3).

Summary

Study 1 hypotheses were supported in that, for the one- and two-interacting predictor conditions considered, a researcher can compute predicted trajectories from either a fitted RCGM or LCGM, regardless of the true distribution of individual differences, and anticipate little bias. At large Ns, predicted trajectories tend to be slightly more efficient when fitting the true generating model. At smaller Ns, however, there is not a consistent efficiency advantage associated with fitting the true generating model. Given documented difficulties associated with properly specifying the discrete or continuous nature of individual differences in practice (e.g., Bauer, 2007), a researcher might be seeking a model that, even when incorrect, will have the lowest MSE for predicted trajectories.
FIGURE 3  Study 1 predicted trajectories for two-interacting-predictor models. Note. LCGM = latent class growth model; RCGM = random coefficient growth model.
### Study 1: Mean Squared Error Results

<table>
<thead>
<tr>
<th>N</th>
<th>Fit RCGM</th>
<th>Fit True LCGM</th>
<th>% MSE Increase or Decrease</th>
<th>Difference in Standardized Root MSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>250</td>
<td>.00829</td>
<td>.00968</td>
<td>−14%</td>
<td>−.0052</td>
</tr>
<tr>
<td>500</td>
<td>.00493</td>
<td>.00495</td>
<td>−.04%</td>
<td>−.0001</td>
</tr>
<tr>
<td>1,000</td>
<td>.00284</td>
<td>.00243</td>
<td>+17%</td>
<td>.0028</td>
</tr>
</tbody>
</table>

### Generating Model = RCGM

<table>
<thead>
<tr>
<th>N</th>
<th>Fit RCGM</th>
<th>Fit LCGM</th>
<th>% MSE Increase or Decrease</th>
<th>Difference in Standardized Root MSE</th>
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<tr>
<td>250</td>
<td>.00911</td>
<td>.01159</td>
<td>+27%</td>
<td>.0089</td>
</tr>
<tr>
<td>500</td>
<td>.00472</td>
<td>.00691</td>
<td>+46%</td>
<td>.0103</td>
</tr>
<tr>
<td>1,000</td>
<td>.00222</td>
<td>.00441</td>
<td>+99%</td>
<td>.0137</td>
</tr>
</tbody>
</table>

### Generating Model = LCGM

<table>
<thead>
<tr>
<th>N</th>
<th>Fit RCGM</th>
<th>Fit True LCGM</th>
<th>% MSE Increase or Decrease</th>
<th>Difference in Standardized Root MSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>250</td>
<td>.01637</td>
<td>.01707</td>
<td>−4%</td>
<td>−.0019</td>
</tr>
<tr>
<td>500</td>
<td>.00944</td>
<td>.00875</td>
<td>+8%</td>
<td>.0026</td>
</tr>
<tr>
<td>1,000</td>
<td>.00462</td>
<td>.00362</td>
<td>+28%</td>
<td>.0056</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>N</th>
<th>Fit RCGM</th>
<th>Fit LCGM</th>
<th>% MSE Increase or Decrease</th>
<th>Difference in Standardized Root MSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>250</td>
<td>.01709</td>
<td>.02098</td>
<td>+23%</td>
<td>.0102</td>
</tr>
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<td>500</td>
<td>.00879</td>
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<tr>
<td>1,000</td>
<td>.00450</td>
<td>.00814</td>
<td>+81%</td>
<td>.0167</td>
</tr>
</tbody>
</table>

Note. MSE = mean squared error; LCGM = latent class growth model; RCGM = random coefficient growth model. 

**(a)**[(misspecified model MSE/true model MSE) × 100 – 100]. **(b)**Standardized root MSE = \(\sqrt{MSE/SD(y)}\). In one-predictor models \(SD(y) = 1.406\). In two-interacting-predictor models \(SD(y) = 1.389\). Difference in standardized root MSE is for (misspecified model – true model).

trajectories at modest \(N\). In this regard, RCGM is slightly preferable to LCGM. The major finding, however, is that both models can do about equally well at recovering complex predictive relationships.

**STUDY 2: ARE THERE DATA CONDITIONS UNDER WHICH ONLY ONE MODEL (LCGM OR RCGM) CAN ACCURATELY RECOVER PREDICTIVE RELATIONSHIPS?**

Study 1 demonstrated that fitting either LCGM or RCGM could recover the same predicted trajectories with little bias, regardless of generating model. However, the focus of Study 1’s generating conditions was neutrality (the reversibility conditions). It is possible that greater bias and/or MSE differences could be observed under more extreme data conditions targeted to pose more difficulty for one fitted model in particular. Study 2 is a generalizability study for the robustness of predicted trajectories to specific suboptimalities that might be encountered in the real world where the true nature of individual differences is unknown.

**Hypotheses**

Predictor relations most difficult for RCGM to recover, and do so parsimoniously, could involve nonlinear, higher order interactions in a generating LCGM. These interactions can be implied by nonmonotonic conditional class probabilities and highly nonmonotonic growth trajectories.

**Hypothesis 1.** When a generating LCGM has implicitly nonlinear, interacting predictor relationships and highly nonmonotonic growth trajectories, RCGM could incur bias as well as efficiency loss compared to LCGM predicted trajectories. Bias would occur if the RCGM’s...
FIGURE 4  Sampling variance under the Study 1 one-predictor LCGM generating condition at $N = 250$, 500, and 1,000. Note. LCGM = latent class growth model; RCGM = random coefficient growth model.

product terms cannot fully recover the LCGM’s implicit nonlinear interactions.

LCGM might have more difficulty recovering predictor effects from a generating RCGM in which growth factor correlations are not in concert with factor-specific predictor effects. One example of this situation would be if growth factors are positively correlated in the population RCGM, but predictor effects have different signs for different growth factors (e.g., Bauer & Curran, 2003a). To compensate for LCGM’s lack of growth-coefficient-specific

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6Because LCGM requires prediction of the entire trajectory as a whole, in this situation LCGM classes could tend to be extracted that have both high, or both low, intercepts and slopes. Then an increment in the predictor could be associated with an increased probability of membership in the high-intercept, high-slope class. Yet, in the population the predictor influences intercepts positively but slopes negatively.
prediction, additional LCGM classes would likely need to be extracted—to indirectly allow more varied predictor–time relations.

**Hypothesis 2.** When a generating RCGM has growth factor correlations not in concert with factor-specific predictor effects, LCGM may incur bias as well as efficiency loss compared to RCGM predicted trajectories. Bias would occur if LCGM cannot support enough classes to adequately approximate growth-factor-specific patterns of predictor relationships.

**Methods**

The Study 2 simulation to test these two hypotheses has 12 cells. For each generating model (LCGM or RCGM) there are 6 cells defined by crossing fitted model (properly specified or misspecified) and \( N = 250, 500, 1,000 \). LCGM and RCGM generating models are intentionally no longer “reversible” as in Study 1 because Study 2’s focus is on tailoring generating conditions to be unfavorable to one model in particular. Software, replications, and starting values paralleled Study 1.

**Population Generating and Fitted Models**

**LCGM with implicitly nonlinear, interacting predictor relationships and highly nonmonotonic growth trajectories.** Although many LCGM applications contain roughly monotonic unconditional class trajectories, as in Study 1, in some applications they are markedly nonmonotonic (e.g., Bushway, Thornberry, & Krohn, 2003; Eggleston, Laub, & Sampson, 2004; Stewart, Livingston, & Dennison, 2008). A modified version of Bushway et al.’s (2003) unconditional class trajectory model of offenses, in Figure 5, was used as our six-class, 20-time-point generating LCGM in Study 2. Nagin and Tremblay (2005b, p. 101) suggested Bushway et al.’s (2003) trajectory pattern was consistent with a taxonomic distinction between desisters and offenders—referring to the fact that there are three desisting, two persisting, and one unaffected trajectory. Although Bushway et al. included no predictors, we added gene and environmental predictor effects that could be consistent with a taxonomic distinction. In the absence of the gene, the effect of risk has similar, severe effects for the two persister trajectories, and similar, milder effects for the three desister trajectories. In the presence of the gene, however, the effects of risk on aggression are suppressed for those in a desister class, but not for those in a persister class. The conditional class probabilities (in Online Appendix Table A.5), and highly nonmonotonic unconditional trajectories in Figure 5 imply highly nonlinear interactions in this generating LCGM:

![FIGURE 5 Study 2 highly nonmonotonic unconditional latent class growth model trajectories (simulated after Bushway et al., 2003). Note. Class 1 = Low; Class 2 = Early-desisters; Class 3 = Middle-desisters; Class 4 = Late-desisters; Class 5 = Medium-chronic; Class 6 = High-chronic. Figure 5 is based on generated data using parameters from Bushway et al. (2003), with the exception that two nearly identical, low-stable classes were collapsed into one, and time scores were recentered and rescaled.

\[
\begin{align*}
\gamma_n|c_i=k & = \gamma_{00}^{(k)} + \gamma_{10}^{(k)}time_n + \gamma_{20}^{(k)}time_n^2 + \gamma_{30}^{(k)}time_n^3 + \epsilon_{ni} \\
\pi_i^{(k)} (r_i, g_i) & = \frac{\exp(\delta_{0i}^{(k)} + \delta_{1i}^{(k)}r_i + \delta_{2i}^{(k)}g_i + \delta_{3i}^{(k)}r_ig_i)}{\sum_{k=1}^{K} \exp(\delta_{0i}^{(k)} + \delta_{1i}^{(k)}r_i + \delta_{2i}^{(k)}g_i + \delta_{3i}^{(k)}r_ig_i)} \tag{15}
\end{align*}
\]

Here \( \epsilon_{ni} \sim N(0, \sigma^2) \). Online Appendix Table A.5 lists all generating parameters. Distributions of \( r_i \) and \( g_i \) were the same as in Study 1, but \( r_i \) was now grand mean centered. Data generated from the Equation 15 LCGM were fit either with the true six-class LCGM or the following RCGM:

\[
\begin{align*}
\gamma_n = \gamma_{00} + \gamma_{10}r_i + \gamma_{02}g_i + \gamma_{03}r_ig_i + \gamma_{04}r_i^2 + \gamma_{05}r_i^2g_i + u_{0i} + (\gamma_{11} + \gamma_{12}g_i + \gamma_{13}r_ig_i + \gamma_{14}r_i^2 + \gamma_{15}r_i^2g_i + u_{1i})time_n + (\gamma_{20} + \gamma_{21}r_i + \gamma_{22}g_i + \gamma_{23}r_ig_i + \gamma_{24}r_i^2 + \gamma_{25}r_i^2g_i + u_{2i})time_n^2 + (\gamma_{30} + \gamma_{31}r_i + \gamma_{32}g_i + \gamma_{33}r_ig_i + \gamma_{34}r_i^2 + \gamma_{35}r_i^2g_i + u_{3i})time_n^3 + \epsilon_{ni} \tag{16}
\end{align*}
\]

where \( \epsilon_{ni} \sim N(0, \sigma^2) \) and \( u_i \sim N(\mathbf{0}, \mathbf{T}) \). In Equation 16 nonlinear terms involving the risk predictor and the risk \( \times \) gene interaction were used to try to capture nonlinearities induced by nonmonotonic effects of predictors on class, and nonmonotonic descriptive trajectories. At 35 free parameters, this RCGM is still parsimonious relative to the generating LCGM (with 45 free parameters).
RCGM with factor-specific predictor relations not in concert with growth factor correlations. The Study 2 generating RCGM is:

\[ y_{it} = \gamma_{00} + \gamma_{01}s_i + \gamma_{02}e_i + \gamma_{03}a_i + \gamma_{04}h_i + u_{0i} + \]

\[ (\gamma_{10} + \gamma_{11}s_i + \gamma_{12}e_i + \gamma_{13}a_i + \gamma_{14}h_i + u_{1i})time_{ti} + (\gamma_{20} + u_{2i})time_{ti}^2 + \epsilon_{it} \]

where \( \epsilon_{it} \sim N(0, \sigma^2) \) and \( u_i \sim N(0, \mathbf{T}) \). Growth coefficients and signs of growth factor correlations were taken from the antisocial behavior RCGM in Bollen and Curran (2006, chap. 5). The signs of predictor effects and the predictor distributions were taken from the two covariates used by Bollen and Curran as well as two other covariates from the same data set (sex \( s_i \sim \text{BIN}(0.52) \); home emotional environment \( e_i \sim N(9.29, 5.15) \); mother age \( a_i \sim N(25.60, 3.38) \); home cognitive environment \( h_i \sim N(9.13, 5.85) \)). Total growth factor correlations were \[ \begin{bmatrix} 1 & \cdot.24 & 1 & \cdot.43 & 1 \end{bmatrix} \]. Some predictors \((e_i \text{ and } h_i)\) had signs that did not follow from these total correlations. That is, despite the positive correlation between intercepts and linear slopes, \( e_i \) had a negative effect on intercepts and a positive effect on linear slopes, whereas \( h_i \) had a negative effect on intercepts and a positive effect on linear slopes.\(^7\) Other predictors’ effects were of the same sign for both intercepts and linear slopes. No predictors affected the quadratic slope. Parameter values for the generating RCGM in Equation 17 are in Online Appendix Table A.6. Data generated from the Equation 17 RCGM were either fit with the true RCGM or the following LCGM, with various numbers of classes:

\[ y_{it} | c, \pi(k) = \gamma_{00}^{(k)} + \gamma_{10}^{(k)}time_{ti} + \gamma_{20}^{(k)}time_{ti}^2 + \epsilon_{it} \]

\[ \pi_i^{(k)}(s_i, e_i, a_i, h_i) = \frac{\exp(\delta_{0}^{(k)} + \delta_{1}^{(k)}s_i + \delta_{2}^{(k)}e_i + \delta_{3}^{(k)}a_i + \delta_{4}^{(k)}h_i)}{\sum_{k=1}^{K} \exp(\delta_{0}^{(k)} + \delta_{1}^{(k)}s_i + \delta_{2}^{(k)}e_i + \delta_{3}^{(k)}a_i + \delta_{4}^{(k)}h_i)} \]

**Data Analysis**

Data analysis paralleled Study 1, except MSEs are only compared within generating model. Study 2 generating models are intentionally not reversible, and have different marginal y variances. Hence, MSEs are not comparable between them.

**Results**

**Convergence and class selection**

Convergence was \( \geq 93.6\% \). When LCGM was generating, the fitted LCGM had the true \( K \) (i.e., \( K = 6 \)). When RCGM was generating, the fitted LCGM had the best-BIC \( K \), shown in Table 2.

**Bias**

As in Study 1, bias plots are shown here for \( N = 500 \); plots for other \( N \)s are in the Online Appendix. In Figure 6—where LCGM is generating—risk and allele were moderators. In Figure 7—where RCGM is generating—home emotional environment was chosen as the moderator. Because it is a continuous predictor, it yields an infinite number of possible predicted trajectories. We conventionally chose to plot it at \(+1 \sigma\), mean, \(-1 \sigma\); other predictors were held at their means.\(^8\) Solid lines are average model-implied and dashed lines are true predicted trajectories.

When LCGM was generating (condition with complex nonlinear interactions), RCGMs with all desired product and power terms were estimable; if not, this could have been a source of bias. When RCGM was generating (condition with growth factor correlations not in concert with factor-specific predictor effects), sufficiently many classes could be estimated to recover predictive relationships. More classes than needed to minimize BIC were estimable at all \( N \)s. Had few classes been estimable or selected (as can occur at low \( N \); Nylund, Asparouhov, & Muthén, 2007; Tofighi & Enders, 2007) then this could have been a source of bias. That is, the two hypothesized culprits for inducing bias remained at bay when fitting misspecified models in Study 2. Hence, when fitting misspecified models, Figures 6 and 7 show similar or just slightly more bias than in the milder Study 1 conditions. As in Study 1, this bias occurred at extreme predictor values. When fitting correctly specified models, RCGMs never result in bias (Figure 7). But correctly specified LCGMs can result in small bias at modest \( N \) (Figure 6).

**MSE**

Study 2 MSE results are shown in Table 4. At \( N = 250 \), under conditions previously deemed unfavorable to RCGM (complex nonlinear interactions condition), fitting a misspecified RCGM still unexpectedly provided better MSE (but merely by 3%) than fitting a true LCGM. By \( N = 500 \) or 1,000, however, fitting a misspecified RCGM provided the expected increase in MSE (by 19%–47%). This pattern of

\(^7\) Another example of factor-specific predictor relationships that are not in concert with growth factor correlations is as follows. A predictor has all-positive or all-negative effects on growth factors, but growth factors are negatively correlated.

\(^8\) A similar pattern of bias results occurs if each of the other predictors is considered as the moderator (not shown).
results resembled Study 1 and again occurred because of relatively worse efficiency for fitting the true LCGM, particularly at low N (see Online Appendix plot of sampling variance.) Yet, in practical terms, standardized MSE differences in Table 4 remained trivial (e.g., .01 SD(y)), as in Study 1. Now consider the conditions deemed unfavorable to LCGM by prior theory (growth factor correlations not in concert with factor-specific predictor effects). There, at all Ns, fitting a misspecified LCGM provided worse MSE (by 244%–312%) than fitting a true RCGM. Given the little bias in Figure 7, LCGM’s MSE gain for the latter condition is likely due to the inefficiency associated with the many classes (6–13) that were required to compensate for LCGM’s lack of growth-coefficient-specific prediction. (If few classes were extracted, more bias was observed.) These relative MSE differences translate to .05 to .08 SD(y) (see the far right column of Table 4); whether such differences are substantively consequential is considered in the next section.

Summary

Study 2 sought to generalize Study 1’s results to conditions that could be unfavorable for recovering predicted trajectories if a misspecified model were fit. Study 2 conditions had greater potential for inducing bias in predicted trajectories, but this potential was not borne out.

Under conditions widely perceived as unfavorable to methods like RCGM, RCGM’s relevant product and power terms were estimable, and little more bias in predictive relationships was observed than if the true LCGM were fit. Moreover, the fewer parameters required by RCGM gave it an MSE advantage over the true LCGM, particularly at lower N. Under conditions perceived as unfavorable to LCGM, enough LCGM classes were estimable to compensate for the lack of growth-factor-specific predictions, leading to little bias. But as a consequence of extracting many classes (often 8–11), LCGM used many more parameters than the true RCGM, leading to efficiency loss. To communicate the practical meaningfulness of this efficiency loss (the largest documented across Study 1 or 2 conditions), power analyses were conducted for fitted models in this condition at N = 250. For fitted LCGMs, power was conducted for joint (multivariate Wald) tests of the effect of each predictor on class membership as a whole. For fitted RCGMs, power was conducted for joint (multivariate Wald) tests of the effect of each predictor on both intercept and linear growth factors. Note that, in this condition’s RCGM generating model, variance in growth explained by the four predictors differed. Unique variance in intercepts and slopes was explained by sex (16% and 13%, respectively), home emotional environment (5% and 44%), mother age (4% and 7%), and home cognitive environment (.05% and 1%). Hence, power would not be expected to be the same across predictors in a given fitted model. For sex, home emotional environment, and mother age, power for RCGM was 100% and for LCGM (where estimable) was 99% to 100%. For home cognitive environment, power for RCGM was 24% and for LCGM (where estimable)
was 11%. Hence, standardized root MSE differences of .08, from Table 4, translate into trivial to modest power differences for corresponding hypotheses tests, depending on effect sizes.

In sum, even under Study 2 conditions it is possible to recover predicted trajectories on average by fitting growth models that misspecify both the functional form of predictor relationships and distribution of individual differences. Efficiency loss is again slightly greater when misspecifying continua as classes than when misspecifying classes as continua.

**GENERAL DISCUSSION**

Certain kinds of complex interactive, nonmonotonic, or class-specific/class-varying predictor relationships have been considered difficult to recover with power and product terms in RCGMs, unless classes are explicitly extracted, as in LCGM (Connell et al., 2006; Laursen & Hoff, 2006; Moffitt, 2006, 2008; Muthén, 2004; Nagin & Tremblay, 2005b; Segawa et al., 2005). Use of methods that allow recovery and investigation of such relations is central to the increasingly discussed person-oriented research paradigm (Ialongo, 2010; Mun, Bates, & Vaschillo, 2010; Sterba & Bauer, 2010a, 2010b; von Eye, 2010). Some prior research has compared classification-based methods with methods related to RCGMs based on their ability to recover predictive relationships when the true distribution of individual differences is unknown (Bogat, 2009; Magnusson, 1998; von Eye & Bogat, 2006; von Eye et al. 2006). This study overcame important limitations of prior research. Specifically, comparisons of prediction results across fitted models used a non-model-specific metric for comparison and were unconfounded by the nature of individual differences in the generating model.

Despite the misspecifications introduced by crossing fitted and generating models, the same predictor relationships were recovered with either RCGM or LCGM. Predictor relationships that are characterized as class-varying or class-specific using LCGM exhibit some kind of nonlinear effects or interactions with time, which can be specified in RCGM as well. For instance, differential effects of school environment on membership in good or poor reading trajectory classes can likely be recovered with a (possibly nonlinear) interaction of school environment and time in a reading RCGM. The results of this study suggest that predicted trajectories from such an RCGM might very well portray the same effects as an LCGM. Indeed, even for high-risk, low-prevalence individuals (“superpredators” in Sampson, Laub, & Eggleston, 2004), predicted trajectories could still be recovered with equivalent or slightly lower bias and variability by RCGM.

In practice, just as in the simulations performed here, researchers’ theoretically posited distribution of individual differences might be incorrect. Although such misspecification could interfere with describing the nature of individual differences in change (Sterba et al., 2012), under conditions studied here, it did not necessarily interfere with another concrete and important goal: recovering relationships between individual growth and predictors. Others have similarly emphasized the importance of establishing whether “predictions can be stable even if latent structure is uncertain” (Butler & Louis, 1992, p. 1990; see also Cudeck & Henly, 2003; Raudenbush, 2005).

**Recommendations for Practice**

If a researcher is going to fit one particular model, whether LCGM or RCGM, applying that model in a manner similar to what was done here can maximize the potential to equivalently recover predictor relations. For example, a researcher could retain the best-BIC $K$ for LCGM and include hypothesized higher-order and potentially nonlinear interaction terms for RCGM. Suppose, for interpretational simplicity, researchers select fewer classes in LCGMs than the $K$ preferred by model selection indices (e.g., Beyers & Seiffge-Krenke, 2007; Brame, Nagin, & Tremblay, 2001;
Gross et al., 2009; Petitclerc, Boivin, Dionne, Zoccolillo, & Tremblay, 2009). This might erode the accuracy of estimated predictor effects. Extended simulation results (not shown) evidenced deterioration of predictive relationship recovery as retained K decreases below best BIC K. Likewise, if researchers choose very restrictive predictive relationship specifications for RCGMs (e.g., neither nonlinear terms nor nonlinear and higher order interactions), recovery of predictive relationships could be eroded (von Eye & Bogat, 2006). The onus is on the researcher to build up RCGMs to include realistically complex predictive relationships. Examples given here show potential for this to be done with fewer estimated parameters than typical LCGMs.

Calculation of predicted trajectories is also recommended to aid the goal of synthesizing LCGM and RCGM results across and within studies. This goal has been of increasing interest (Connell et al., 2006; Reinecke, 2006; Romens, Abramson, & Alloy, 2009) because in many research areas such as substance abuse, antisocial behavior, or educational achievement, numerous applications of RCGMs and LCGMs now exist. Predicted trajectories can be obtained from LCGM studies regardless of the number of classes extracted. They can be compared to those from RCGM, across and/or within studies, so long as similar predictor sets were used. Predicted trajectories can be calculated from estimates already provided in published articles, using Equations 8 and 9. It should be reemphasized that Nagin and Tremblay (2005a, pp. 883–885) recommended calculation of predicted trajectories for interpreting conditional LCGMs regardless of whether a direct or indirect interpretation of classes is desired, so they are general in that respect. Our results on recoverability of predicted trajectories across diverse generating and fitted LCGMs and RCGMs provide the first empirical justification for comparing predicted trajectories in practice when the true discrete or continuous nature of individual differences is unknown.

Limitations and Future Directions

Several limitations of this study provide opportunities for future work. First, no time-varying predictors were included; they are often not included in LCGMs. Predicted trajectories involving interactions among time-varying predictors could be plotted as well (see Sterba, in press). Second, we used only conditionally normal repeated measures (the most common outcome in Sterba et al.’s [2012] review); generalizability of results to binary or count outcomes is a topic for future research. Third, one measure of predicted relationship recovery was used that assesses predictive accuracy in the population at large. Predicted trajectories could also be conditioned on individual factor scores (for RCGM) or posterior probabilities of class membership (for LCGM). Fourth, future research could compare bias and efficiency of predicted relationships using growth mixture models, LCGM, and RCGM. Fifth, future studies could consider whether other misspecifications differentially affect predicted relationship recovery in RCGM versus LCGM (e.g., measurement error, or restricted-range predictors; Aguinis, 1995).
Conclusions
This study related and evaluated alternative strategies for predictive relationship recovery in two popular longitudinal models. Counter to current thinking in applied research, both LCGMs and RCGMs were shown to recover predicted trajectories under a variety of different data generating conditions. These conditions implied somewhat different mean structures and distributions of individual differences. In practice, the true generating model is unknown; hence, it is relevant for researchers to note that the anticipated nature of such relationships (even if complex and class-varying) need not preclude use of either model for prediction. Finally, this study provided an empirically supported rationale for synthesizing results on predictor–time relations across studies and across methods, using predicted trajectories.

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REFERENCES


